

Sea ice stability and early warning signals

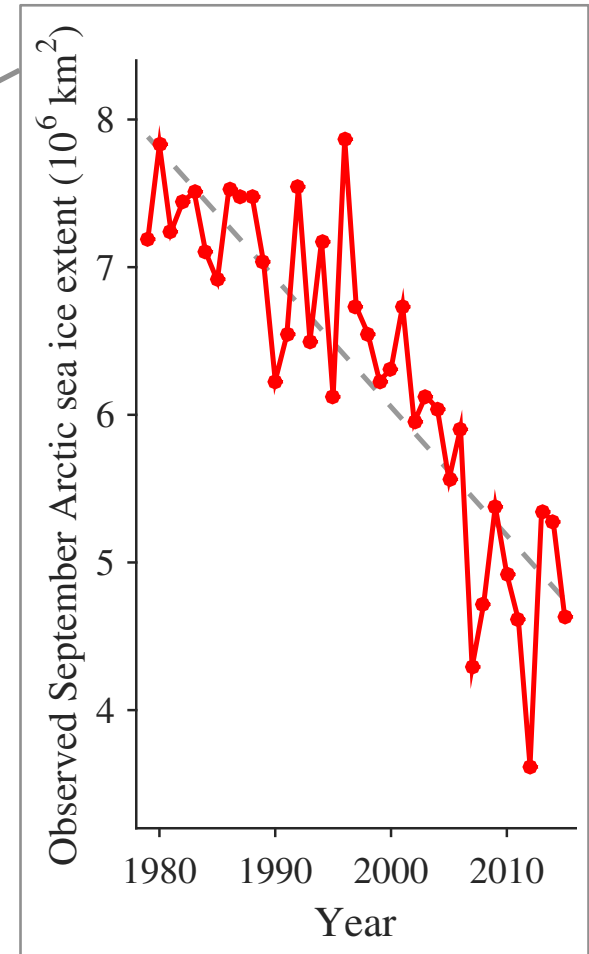
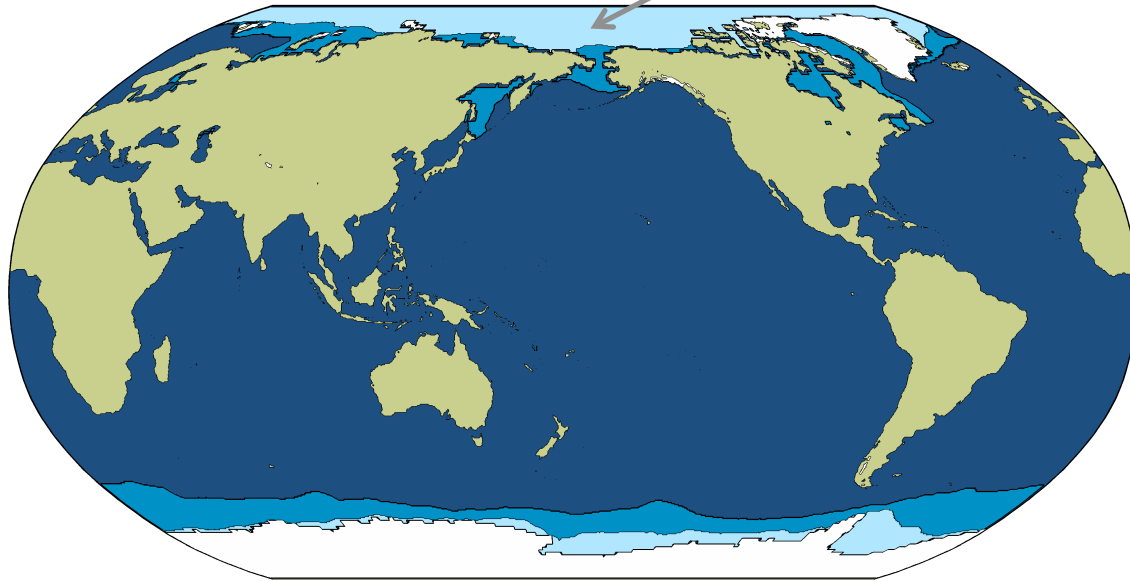
Till Wagner and Ian Eisenman

**Scripps Institution of Oceanography
University of California San Diego**

Part 1

Sea ice stability

Recent Arctic sea ice retreat

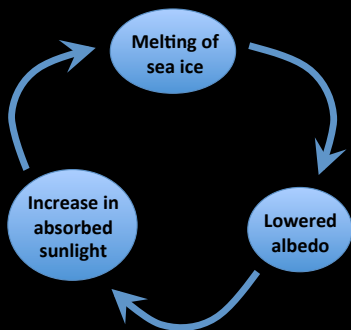



- Arctic summer sea ice extent **diminished by 45%** during 36-yr satellite era (1979-2014).

Ice-albedo feedback

- The difference in albedo between ice and ocean causes a **positive feedback**.
- According to satellite measurements, sea ice retreat caused the solar energy input into the Arctic to increase by $6.4 \pm 0.9 \text{ Wm}^{-2}$ during 1979-2011.
- Contribution to global energy budget is **25% as large as the direct radiative forcing from rising CO₂** ($0.2\text{Wm}^{-2} / 0.8\text{Wm}^{-2}$).

Pistone, Eisenman, & Ramanathan (2014)



darker  no change

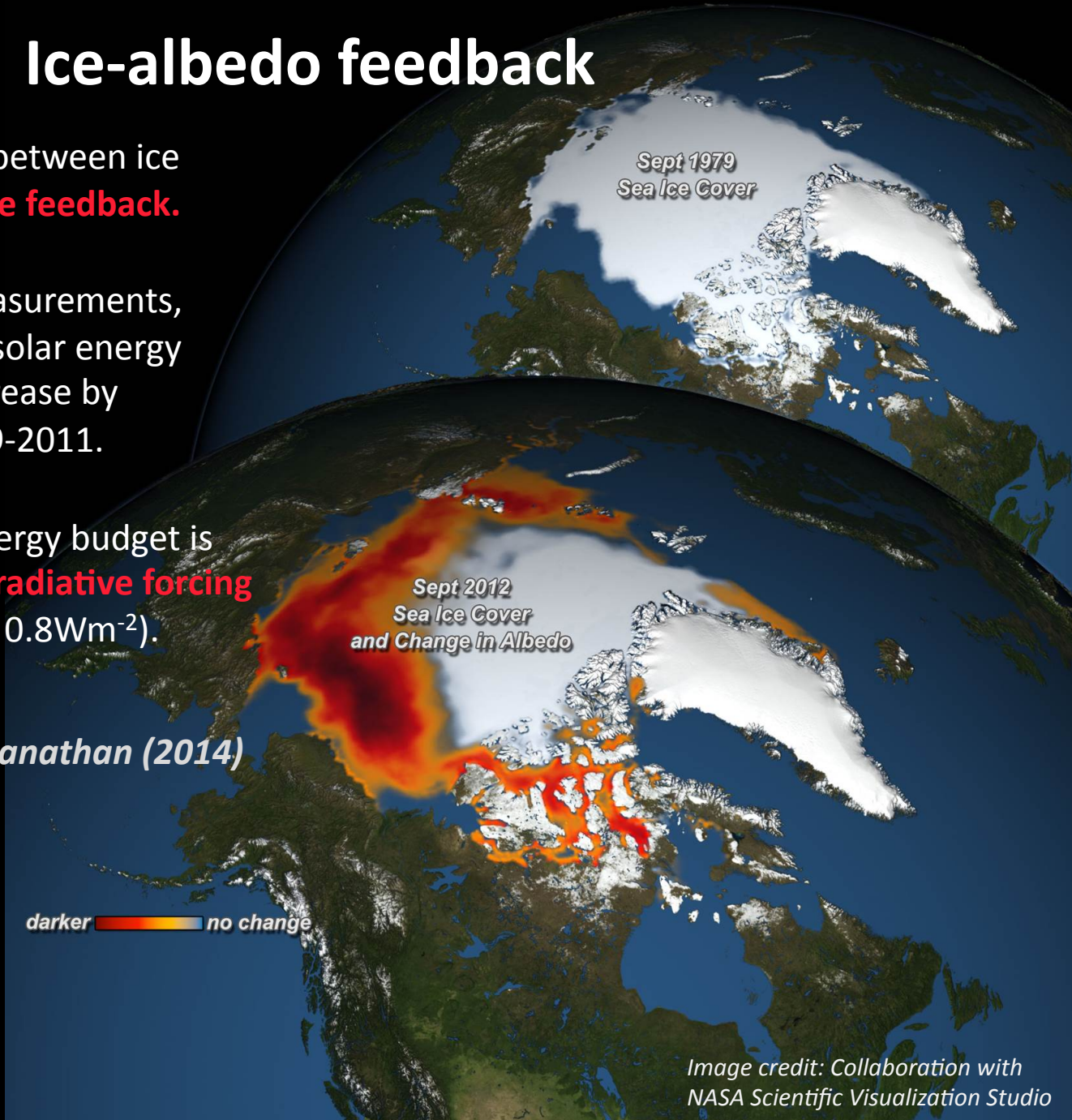


Image credit: Collaboration with NASA Scientific Visualization Studio

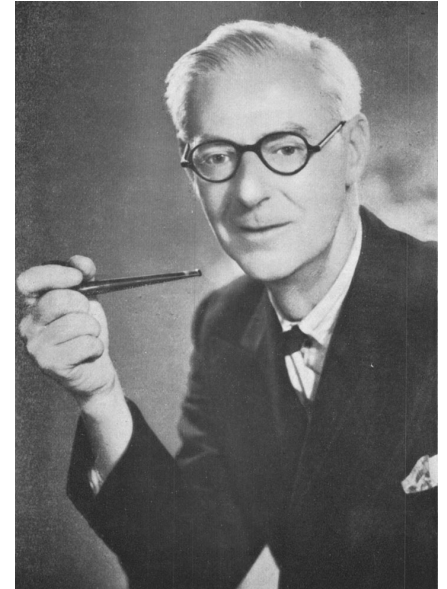
Instability from the ice-albedo feedback: early history



James Croll (1821-1890)

- Croll (1875) was first to identify the importance of ice albedo as a positive feedback (in context of glacial cycles).

- Brooks (1926) argued that the ice-albedo feedback would allow two stable climate states: one with little ice, another with a vast white polar ice cap.



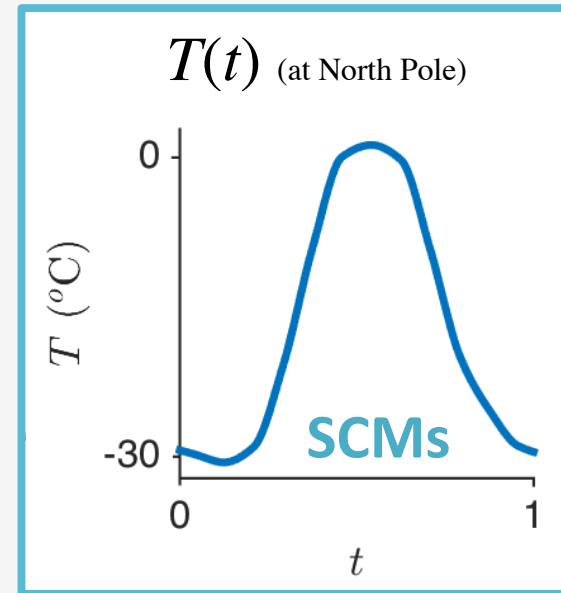
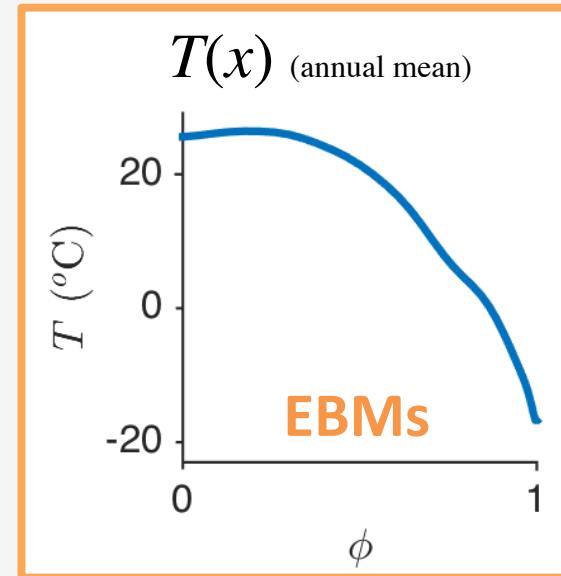
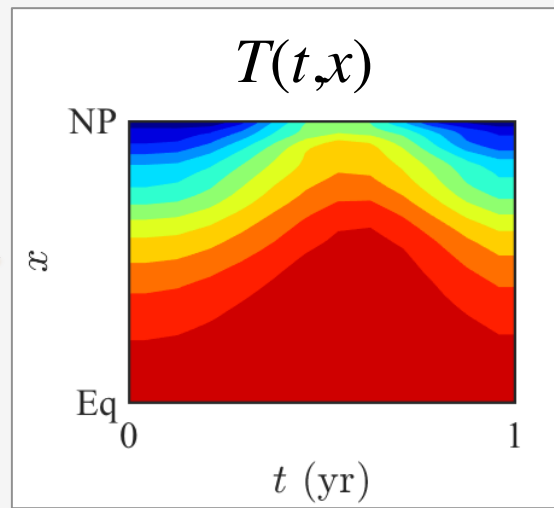
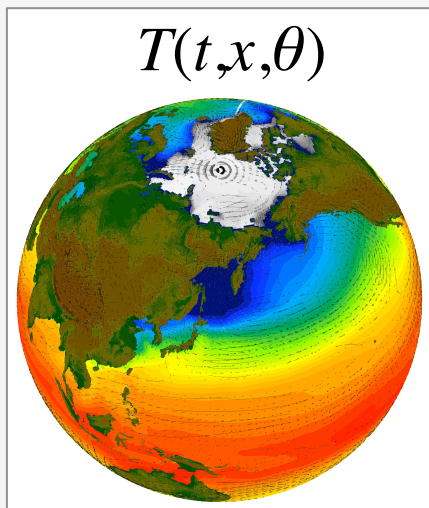
C.E.P. Brooks (1888-1957)



Mikhail Budyko (1920-2001)

- Budyko (1966) used an energy budget estimate to argue that if the sea ice were removed from the Arctic today then it would not return due to the ice-albedo feedback.

Models of ice albedo and climate



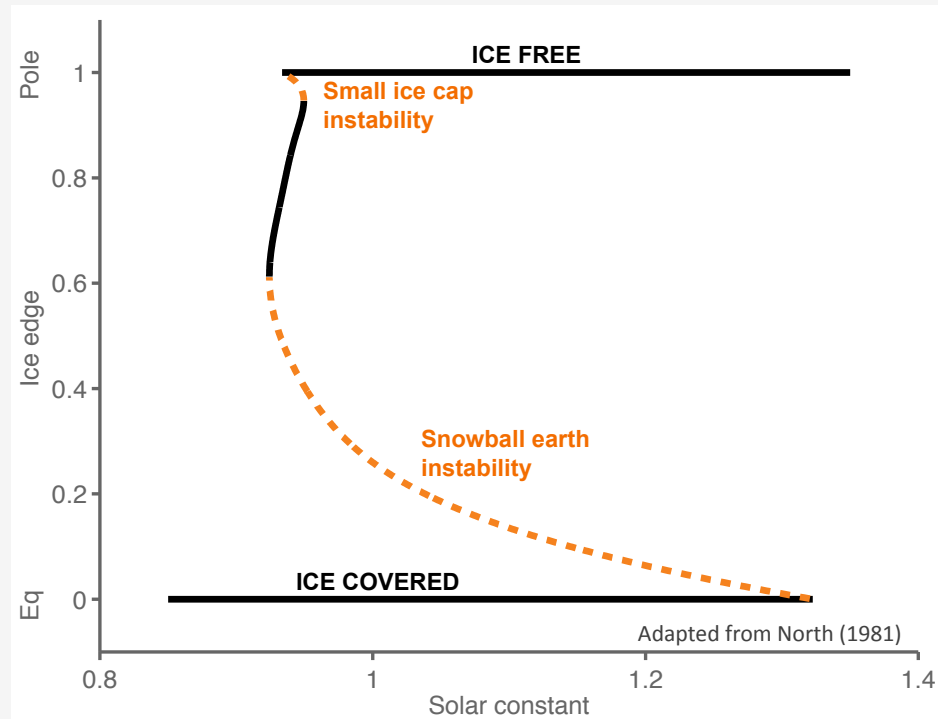
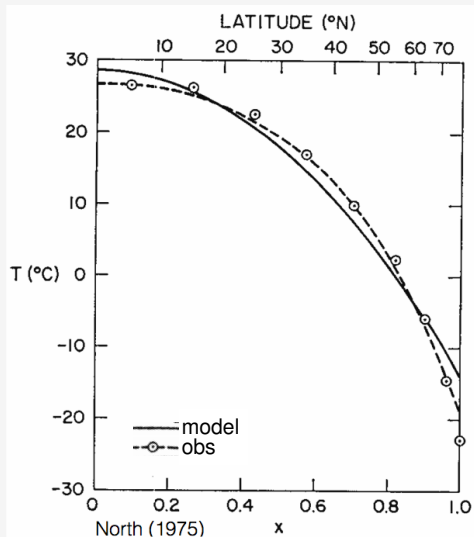
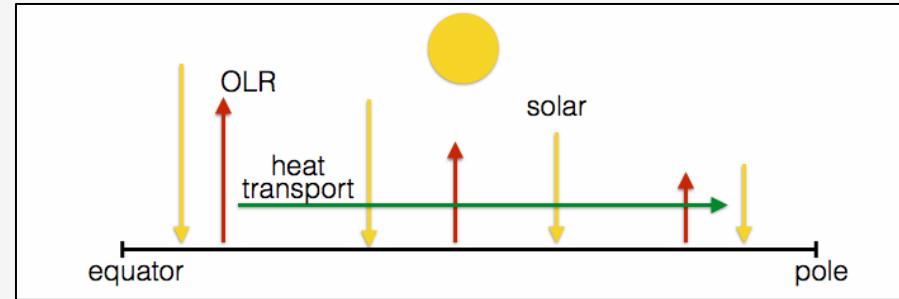
• Two simplest idealized model approaches:

1) Annual-mean $T(x)$: Energy Balance Models (EBMs)

2) North Pole $T(t)$: Single Column Models (SCMs)

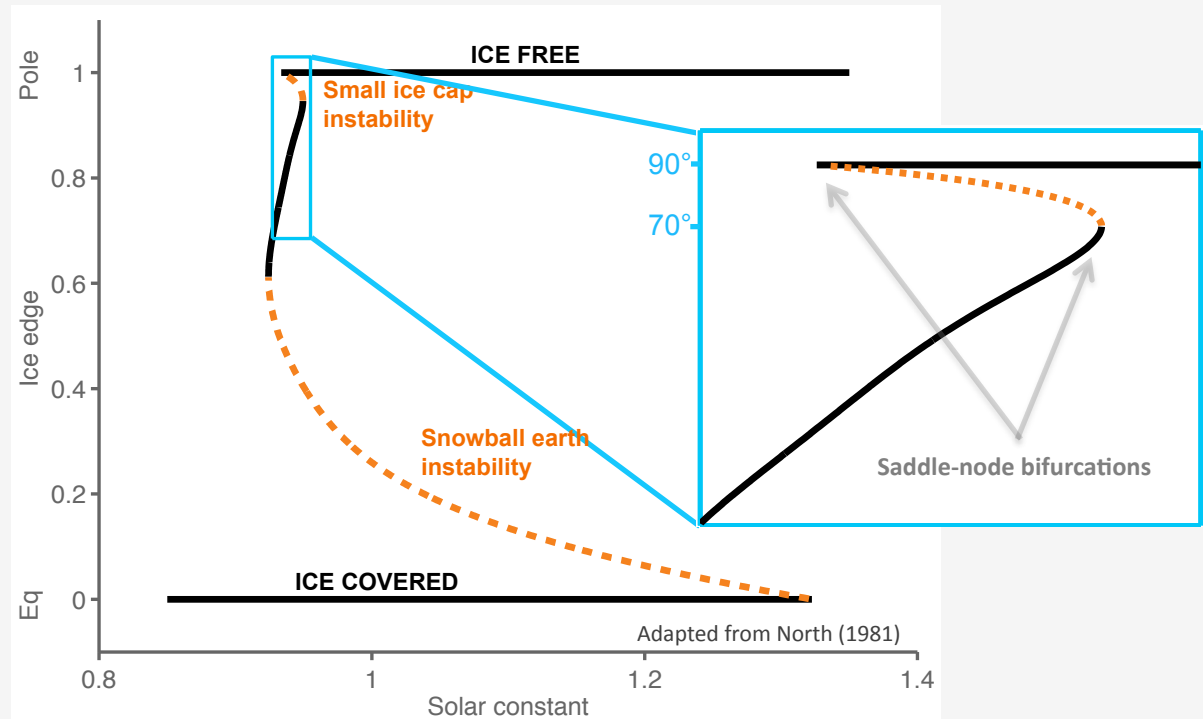
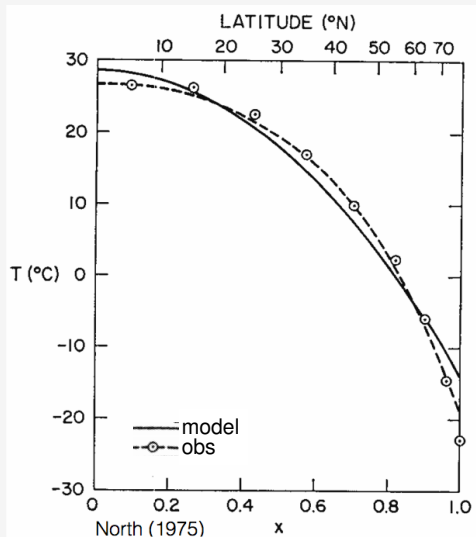
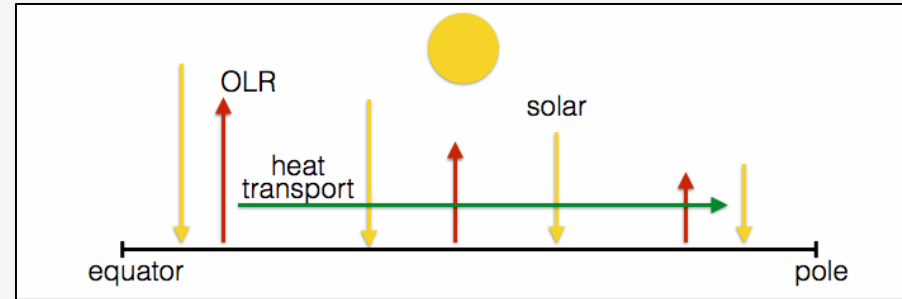
Energy Balance Models (EBMs)

- Most classic type of global climate model (Budyko 1969, Sellers 1969).
- Albedo depends on T . Heat transport as $D\nabla^2 T$.
- Resulting $T(x)$ agrees with observations.
- **Simulates instability and hysteresis during sea ice retreat** (Budyko 1969; Held & Suarez 1974; Lindzen & Farrell 1977; Suarez & Held 1979; North 1975ab, 1981, 1984, 1991; Winton 2008; Rose & Marshall 2009).



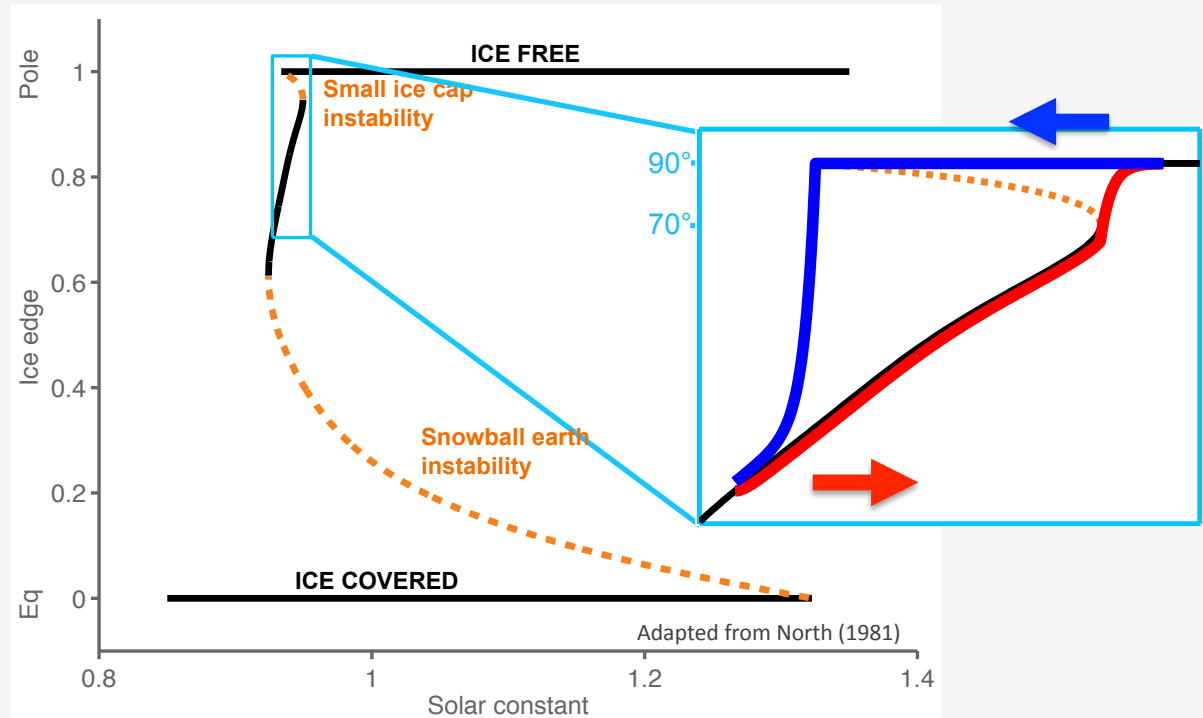
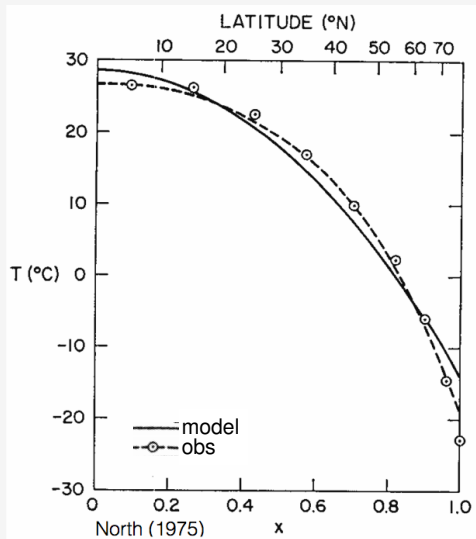
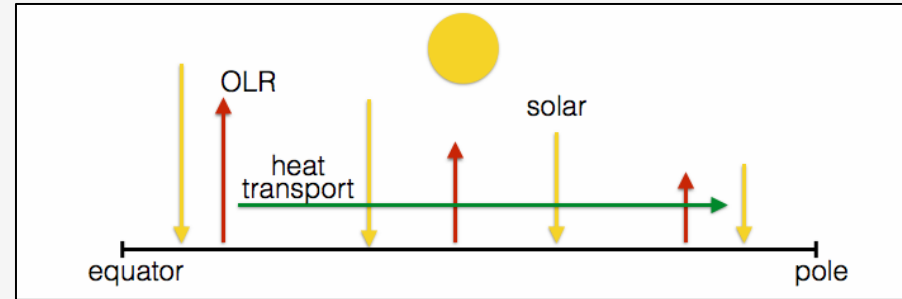
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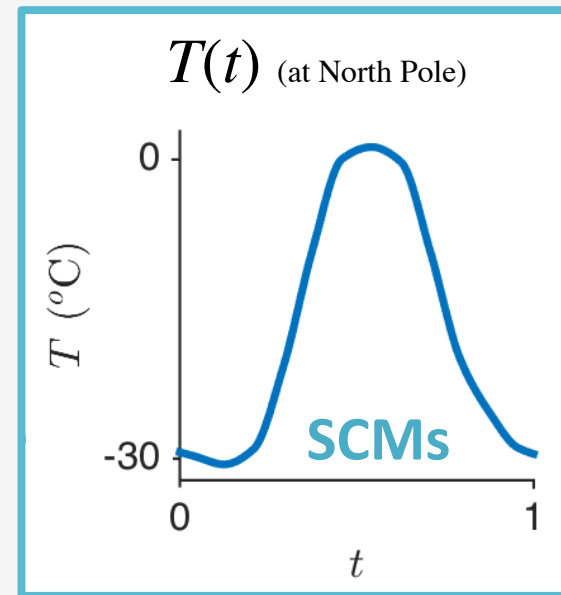
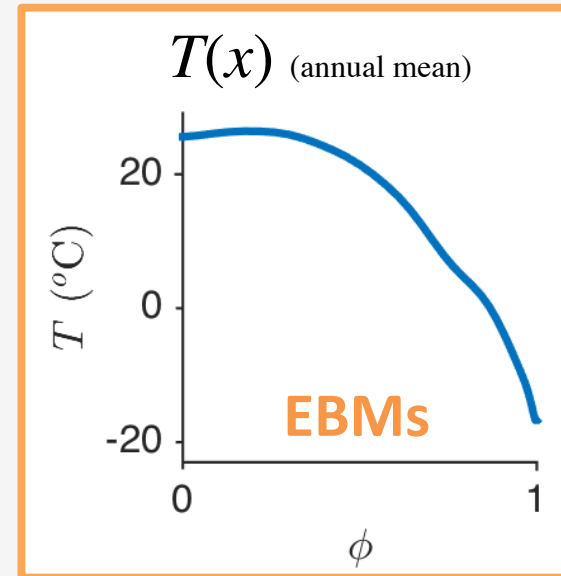
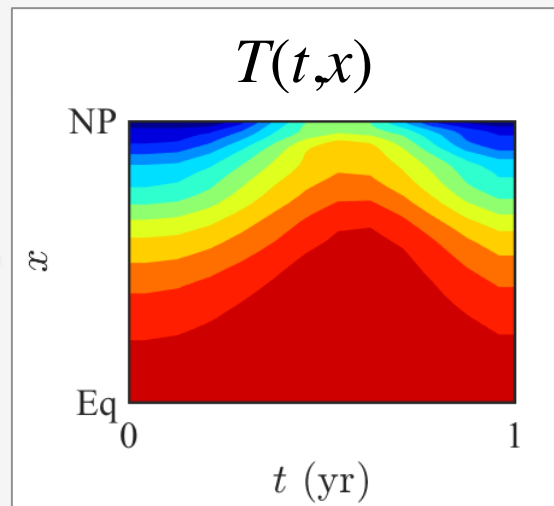
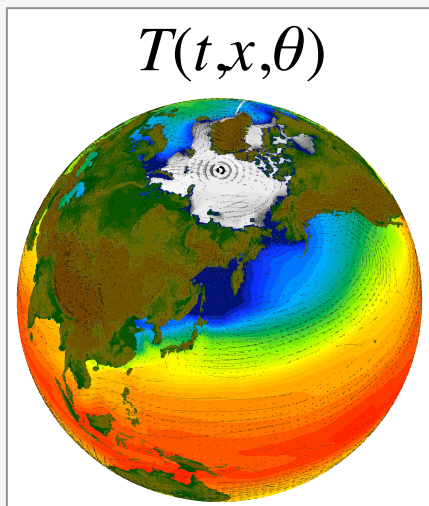


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Models of ice albedo and climate



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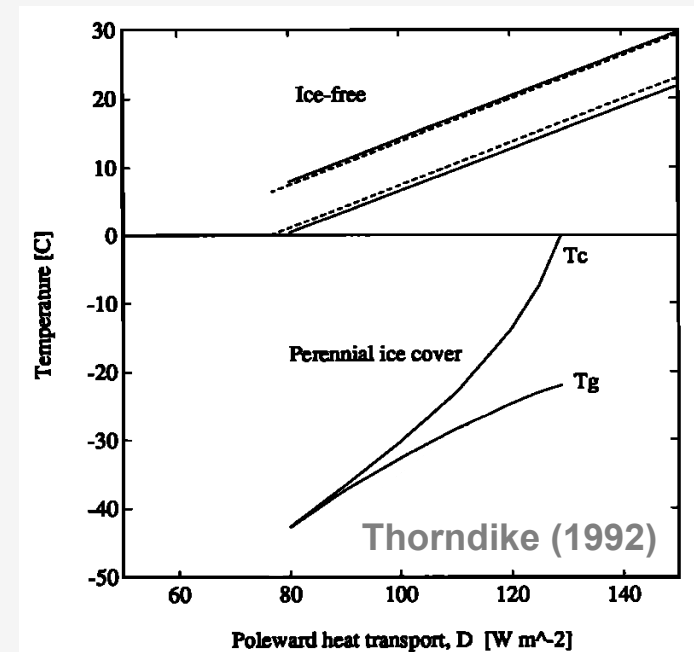
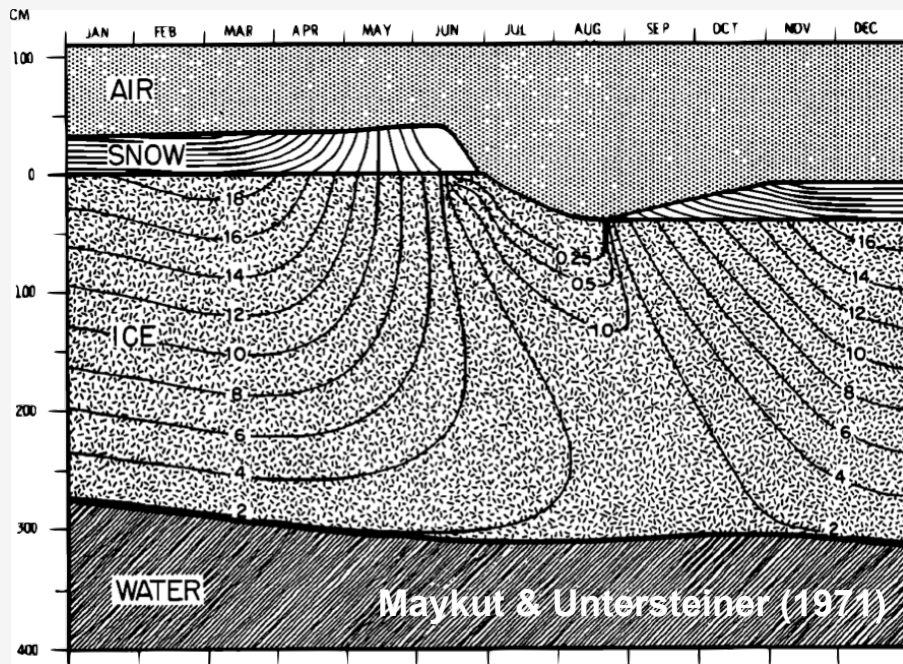
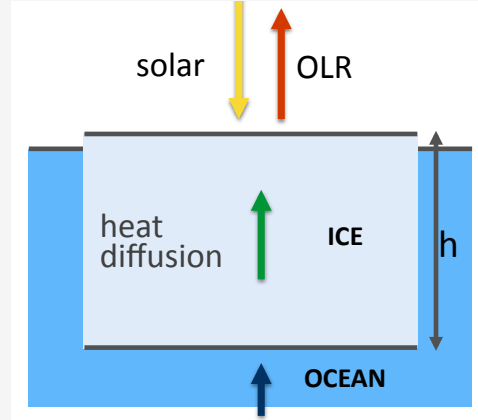
2) North Pole $T(t)$: Single Column Models (SCMs)

Single Column Models (SCMs)

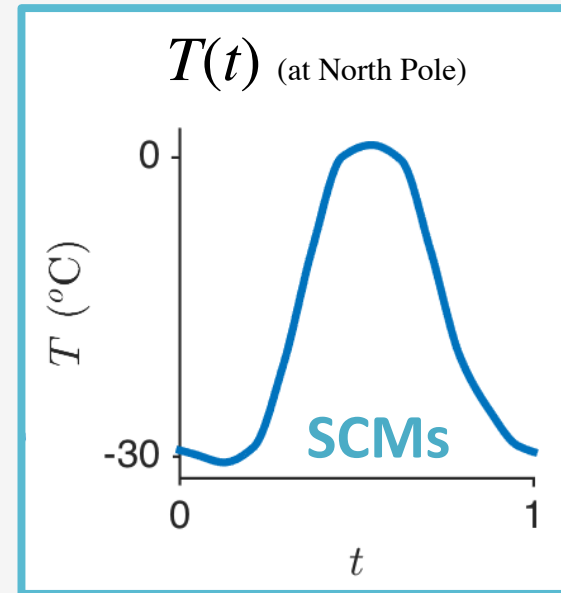
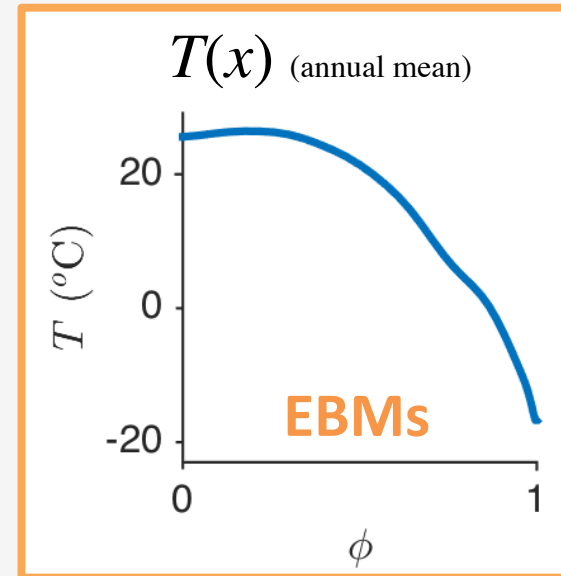
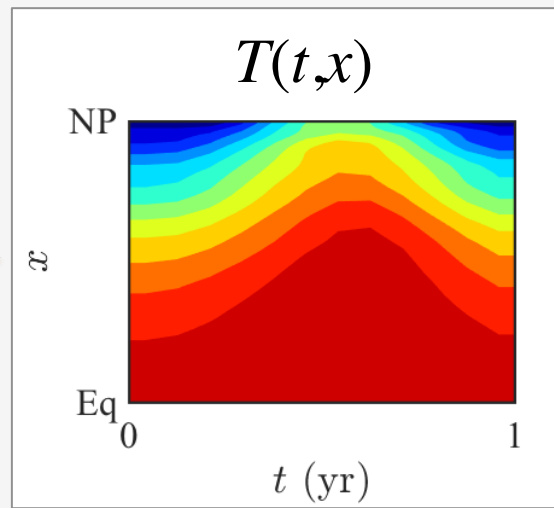
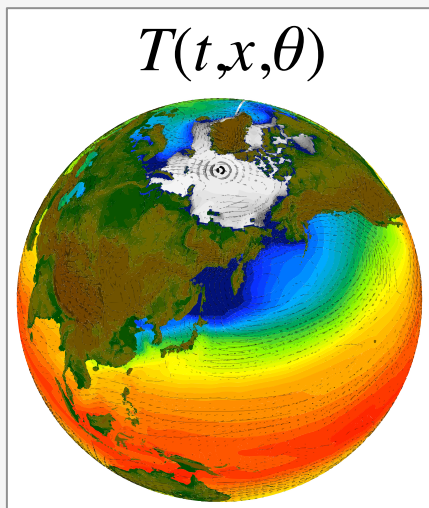
- Classic method to study Arctic sea ice (Maykut & Untersteiner 1971).
- Includes seasonal cycle and sea ice thermodynamic processes.
- Resulting $T(t)$ & $h(t)$ agree with observations.

- **Also simulates instability and hysteresis during sea ice retreat**

(Thorndike 1992; Flato and Brown 1996; Björk 2002; Eisenman 2007, 2012; Eisenman and Wettlaufer 2009; Müller-Stoffels and Wackerbauer 2011, 2012; Abbot et al. 2011; Moon and Wettlaufer 2011, 2012; Björk et al. 2013).



Models of ice albedo and climate

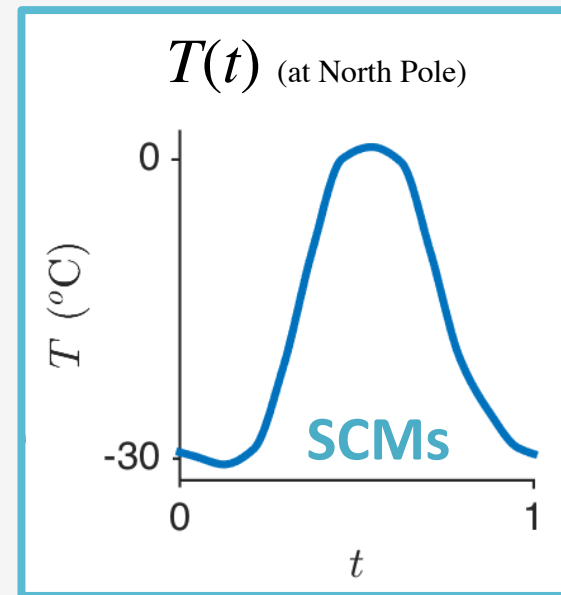
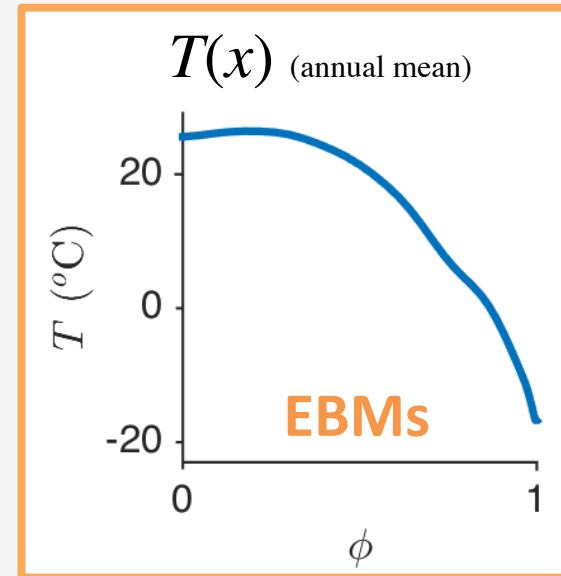
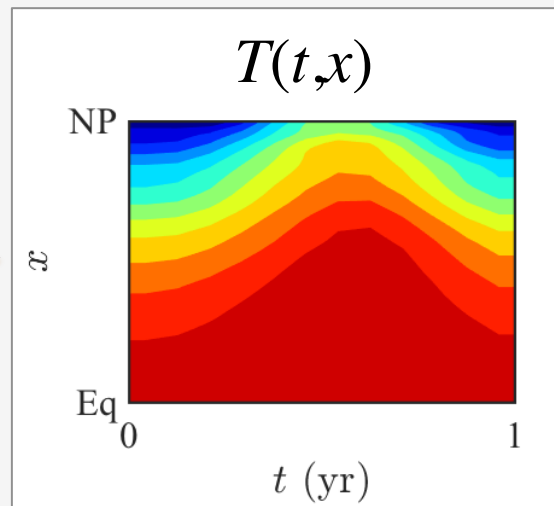
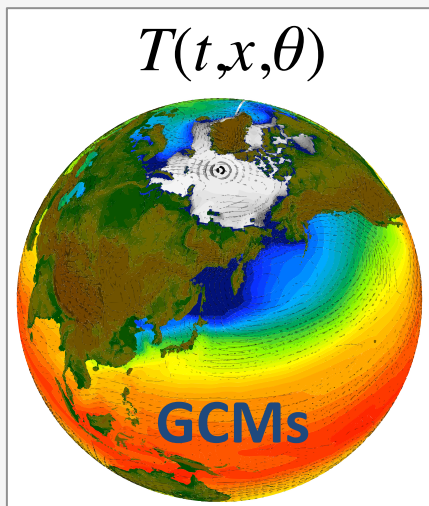


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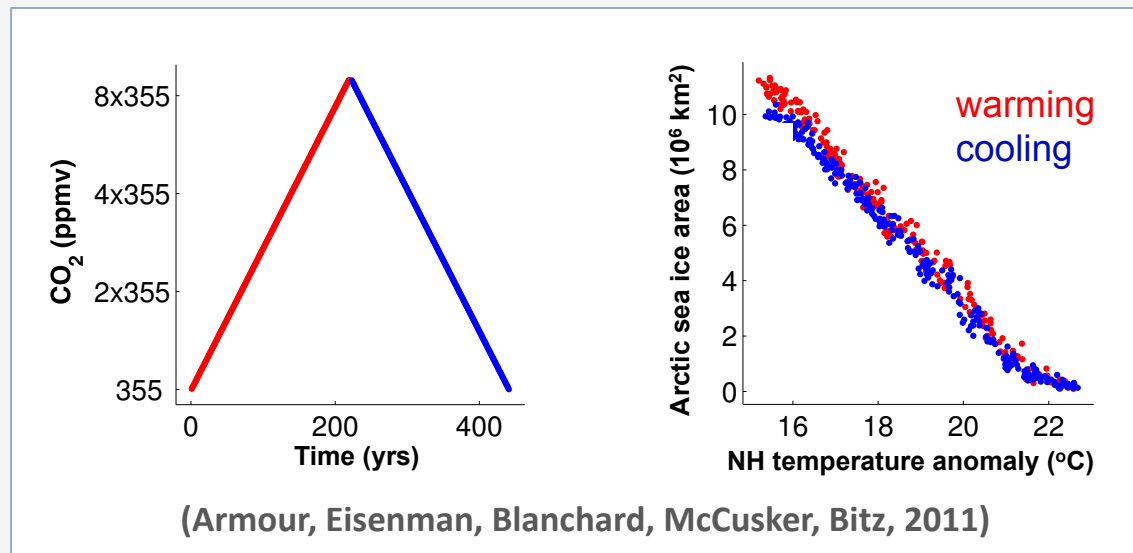
Models of ice albedo and climate



- Two simplest idealized model approaches:
 - 1) Annual-mean $T(x)$: Energy Balance Models (**EBMs**)
 - 2) North Pole $T(t)$: Single Column Models (**SCMs**)
- Comprehensive global climate models (**GCMs**).

Comprehensive global climate models (GCMs)

- State-of-the art climate models, including atmosphere, ocean, sea ice, and land surface.
- NCAR CCSM3 (a GCM) **does not simulate instability and hysteresis** when **CO₂ increased** until all ice gone and **then decreased** back to starting point (Armour et al. 2011).
 - Behavior is **strikingly linear** – no hint of nonlinearity from ice-albedo feedback.



- Similar simulations with **other GCMs yield similar results** (e.g., Ridley et al. 2012; Li et al. 2013; cf. Ferreira et al. 2011).
- Using less direct approaches, a range of GCMs were found not to show evidence of instability from the ice-albedo feedback (Winton 2006, 2008; Ridley et al. 2007; Tietsche et al. 2011).

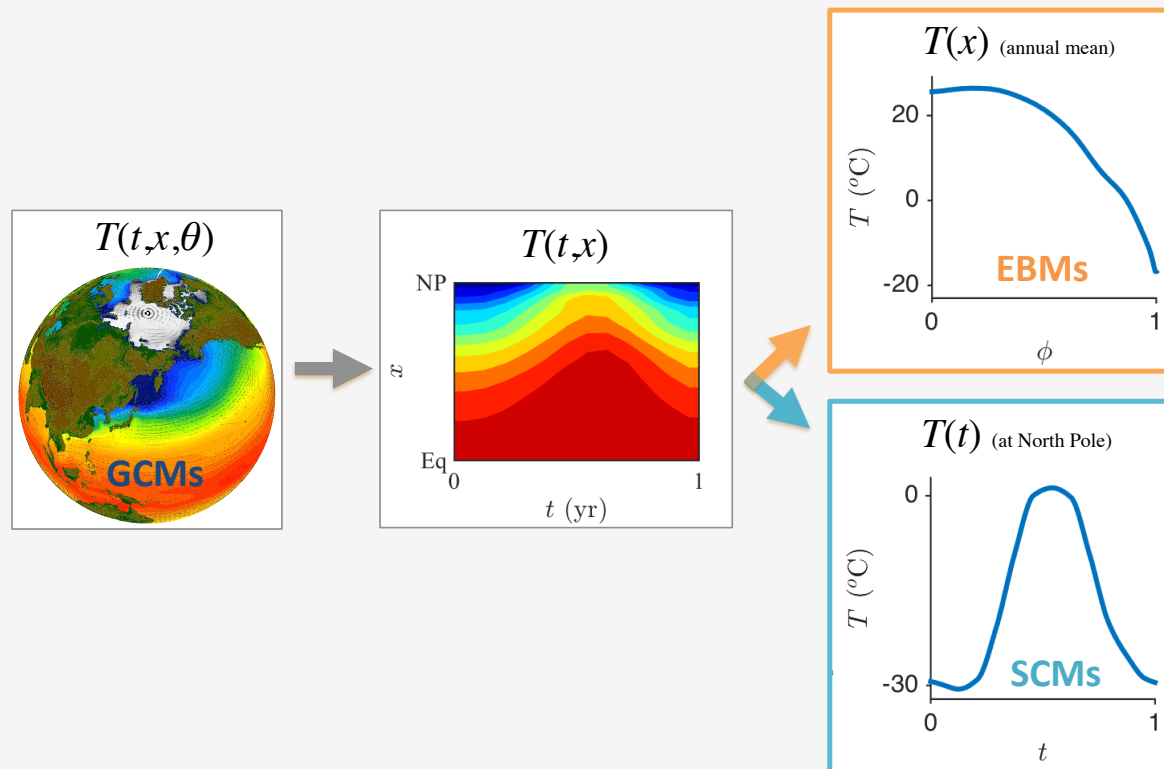
Question

Idealized climate models find instability in the sea ice cover.

Comprehensive climate models do not.

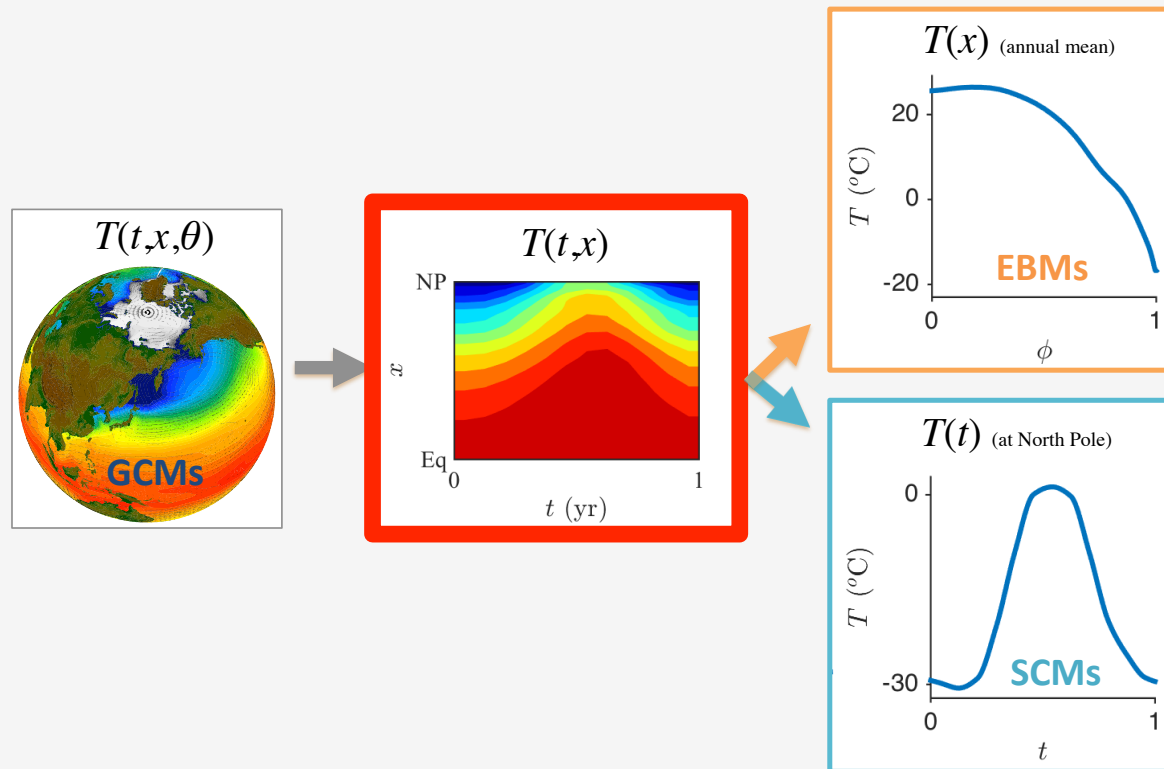
Why?

Do idealized models miss essential physics?
Or do comprehensive GCMs get things wrong?



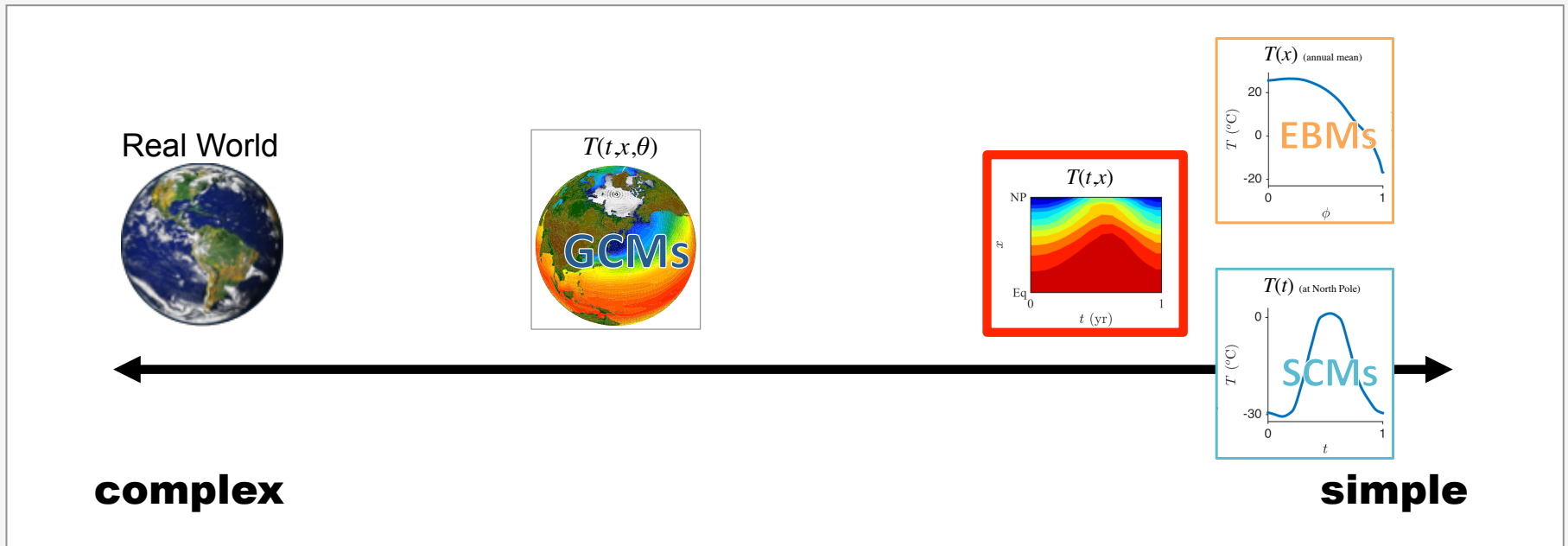
Approach

Construct an idealized model that contains the physics in both EBMs and SCMs: an “EBM-SCM” that simulates $T(t,x)$.

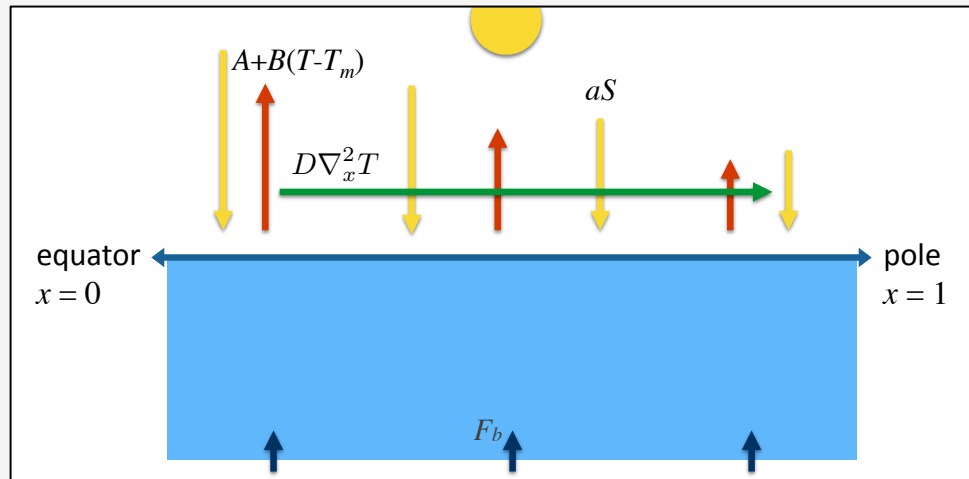


Approach

Construct an idealized model that contains the physics in both EBMs and SCMs: an “EBM-SCM” that simulates $T(t,x)$.



Model development: EBM



ODE for $T(x)$: $0 = f_0 + F$ (scalable climate forcing, F)

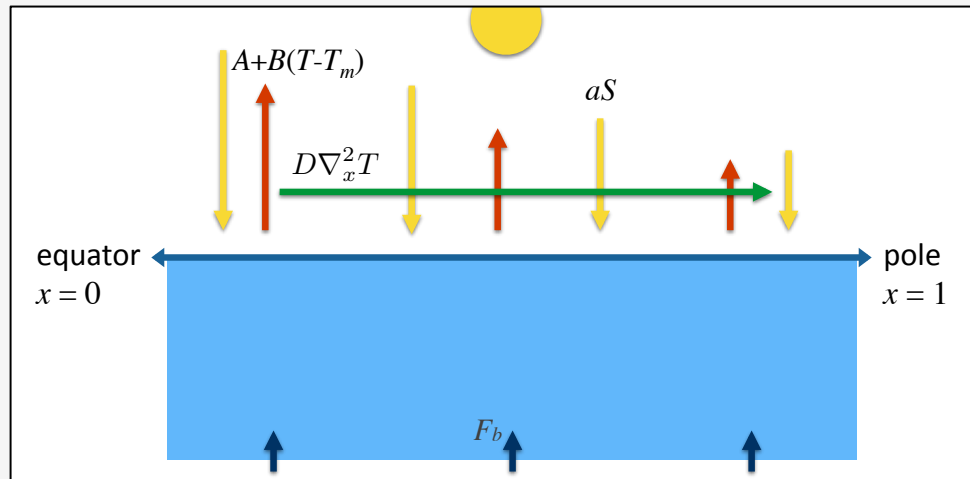
$$f_0 \equiv aS - [A + B(T - T_m)] + D\nabla_x^2 T + F_b$$

Insolation: $S(x) = S_0 - S_2 x^2$

Co-albedo: $a(x, T) = \begin{cases} a_0 - a_2 x^2 & T > T_m \\ a_i & T \leq T_m \end{cases}$

Heat transport: $D\nabla_x^2 T = D \frac{\partial}{\partial x} \left[(1 - x^2) \frac{\partial T}{\partial x} \right]$

Model development: + *Seasonal cycle*



PDE for $T(x, t)$:

$$c_w \frac{\partial T}{\partial t} = f_0 + F$$

(scalable climate forcing, F)

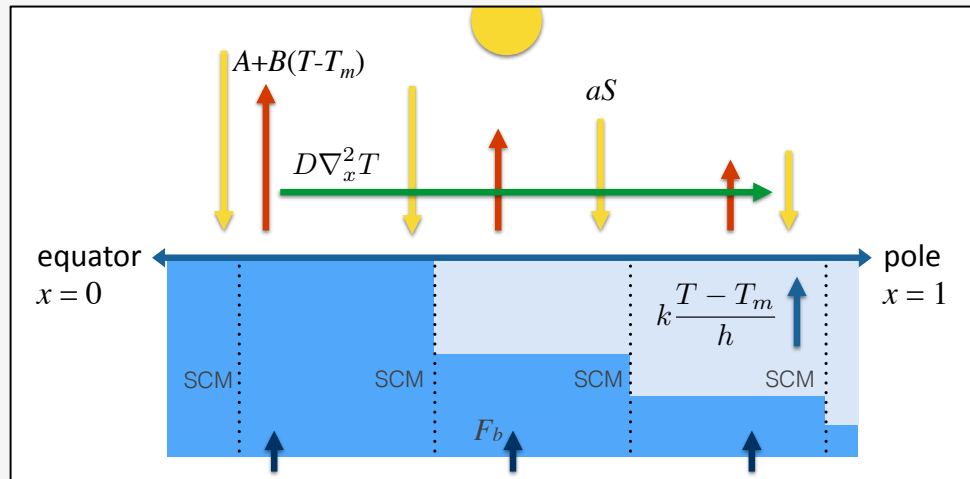
$$f_0 \equiv aS - [A + B(T - T_m)] + D\nabla_x^2 T + F_b$$

Insolation: $S(t, x) = S_0 - S_2 x^2 - S_1 x \cos \omega t$

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Model development: + SCM physics



$$E \equiv \begin{cases} -L_f h & E < 0 \\ c_w (T - T_m) & E \geq 0 \end{cases}$$

(Eisenman & Wettlaufer 2009)

PDE for $E(x, t)$: $\frac{\partial E}{\partial t} = f_0 + F$ (scalable climate forcing, F)

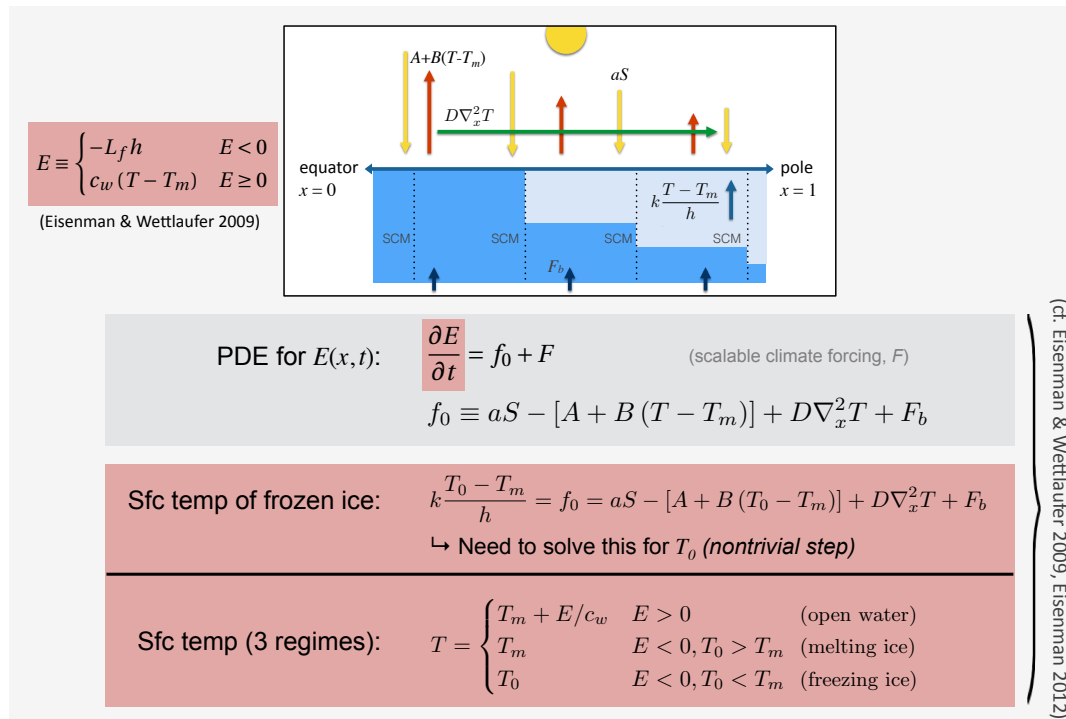
$$f_0 \equiv aS - [A + B(T - T_m)] + D\nabla_x^2 T + F_b$$

Sfc temp of frozen ice: $k \frac{T_0 - T_m}{h} = f_0 = aS - [A + B(T_0 - T_m)] + D\nabla_x^2 T + F_b$

↳ Need to solve this for T_0 (*nontrivial step*)

Sfc temp (3 regimes): $T = \begin{cases} T_m + E/c_w & E > 0 & \text{(open water)} \\ T_m & E < 0, T_0 > T_m & \text{(melting ice)} \\ T_0 & E < 0, T_0 < T_m & \text{(freezing ice)} \end{cases}$

Model development: Numerical solution

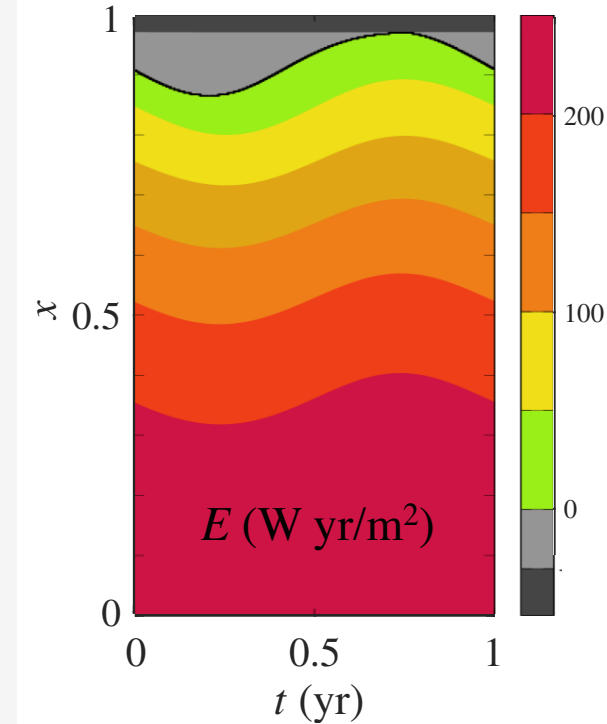
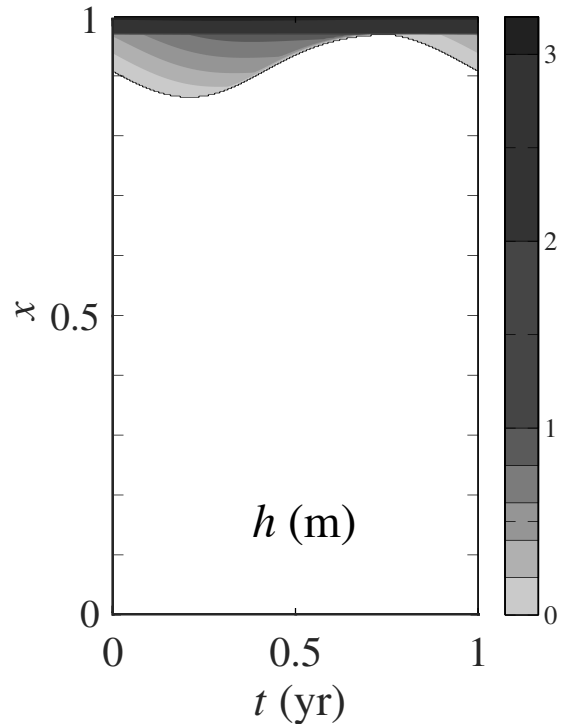
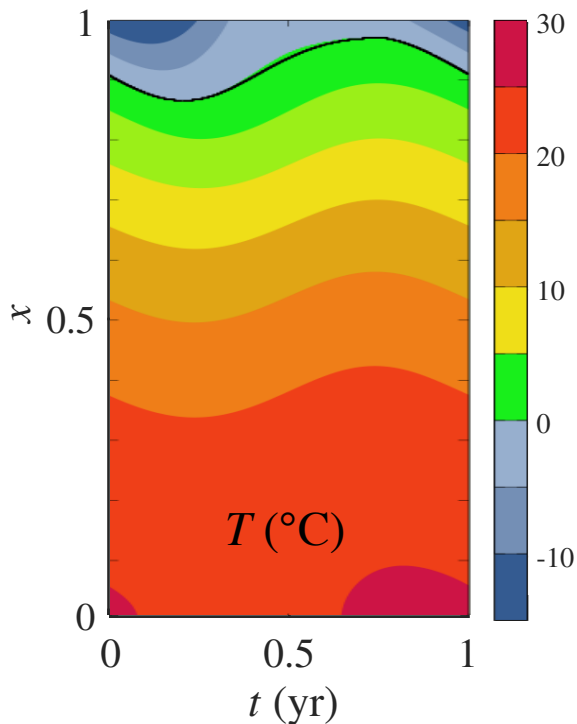


- At each timestep, T_0 is solution of a nonlinear ODE involving $\nabla^2 T$ and a free boundary between melting and freezing ice surfaces.
- Rather than numerically solve this ODE at each timestep, we constructed an analogous two-layer system:
 - Diffusion occurs in “ghost layer” (Implicit Euler time stepping).
 - All other processes occur in main layer (Forward Euler time stepping).
 - Energy exchanged between layers to relax ghost layer temp toward main layer.
 - Two-layer system is equivalent to physical model in limit of fast relaxation time.

$$\frac{\partial E}{\partial t} = f_0 + F$$

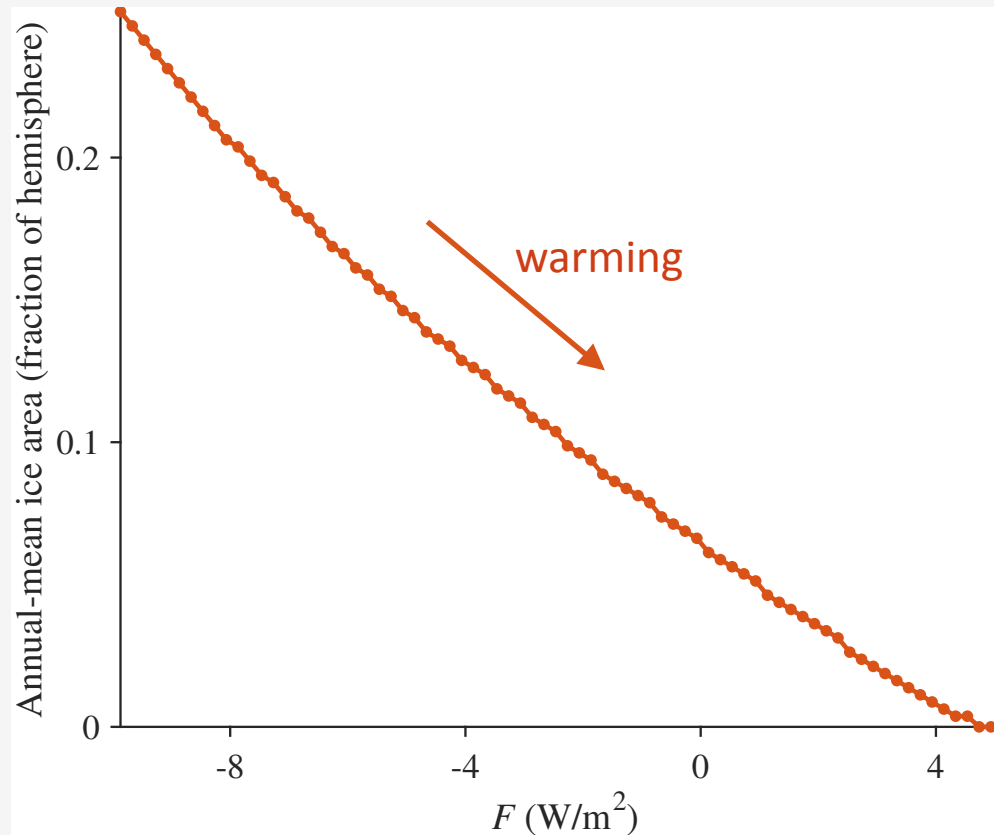
Default model run

- With $F = 0$, spun-up model state **agrees reasonably well with observed current climate**.
 - Ice edge migrates seasonally between 60°N and 80°N, with thick multiyear ice and thin seasonal ice.
- Simulated equilibrium model state is fully determined by spun-up $E(x,t)$ during 1 year.



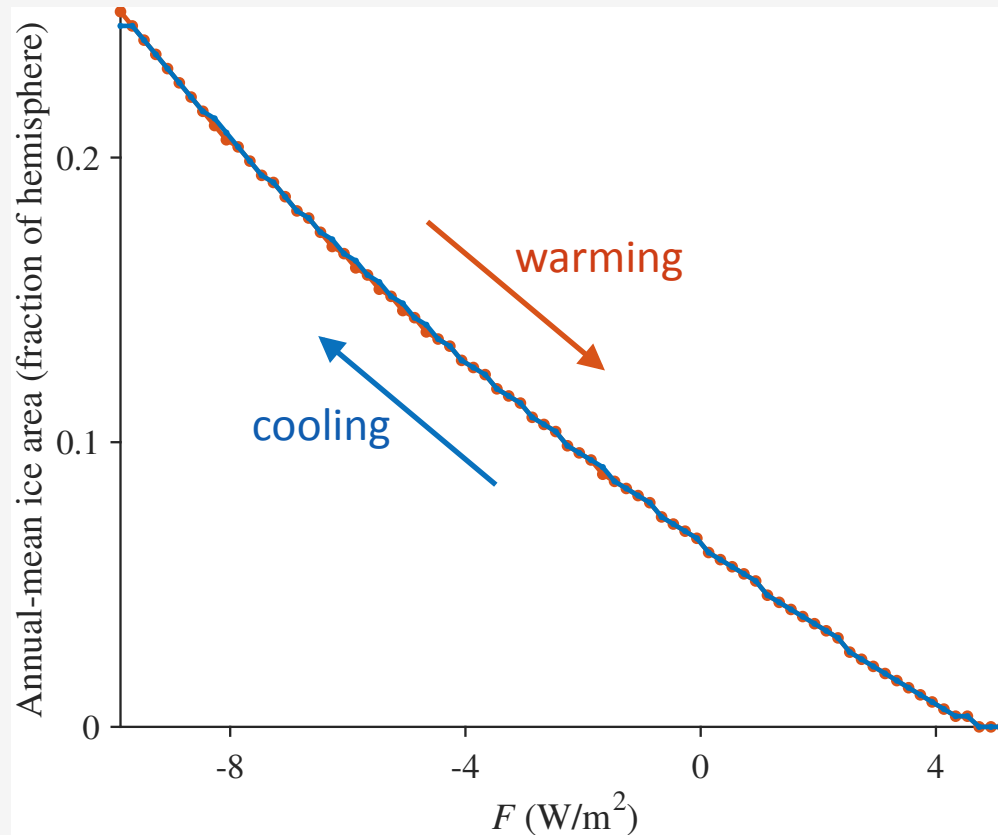
Test for hysteresis

- Sea ice retreat is approximately **linear** when F is ramped up.



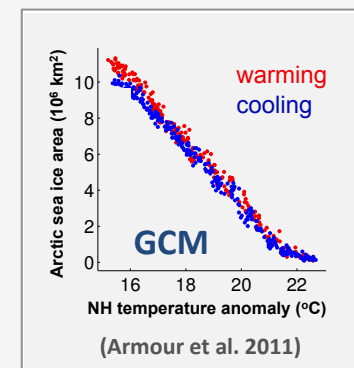
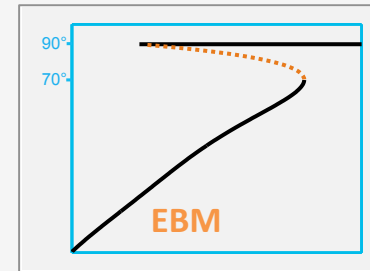
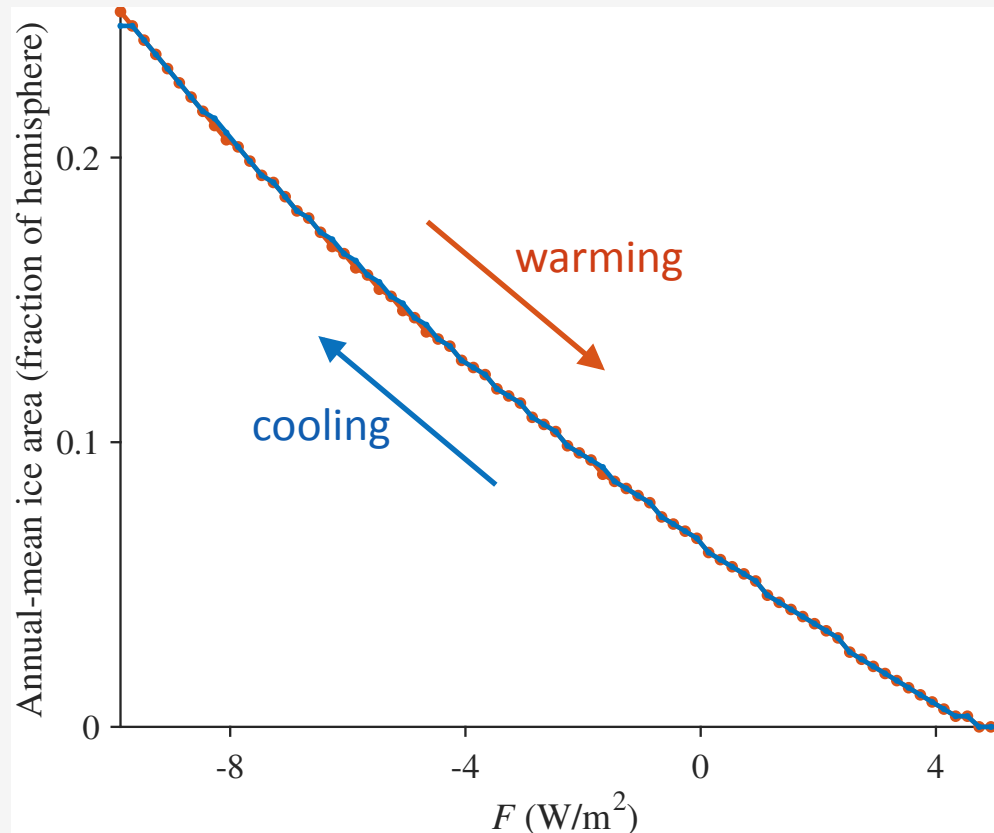
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- Ramping F back down causes ice to recover along identical trajectory.
- **No instability during sea ice retreat in this model!**



Test for hysteresis

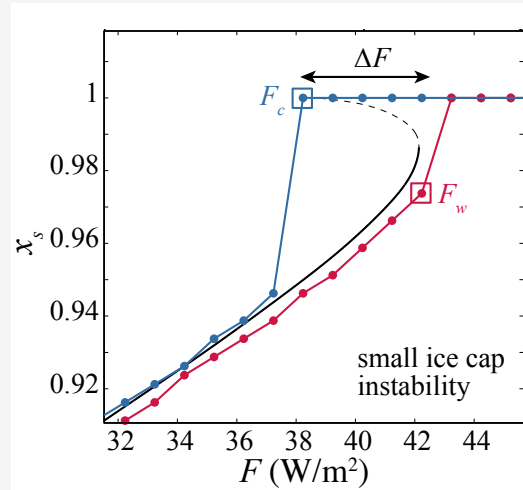
- Sea ice retreat is approximately **linear** when F is ramped up.
- Ramping F back down causes ice to recover along identical trajectory.
- **No instability during sea ice retreat in this model!**
- This EBM-SCM resembles a GCM rather than an EBM or SCM.



Reduction to earlier models

$$S(t, x) = S_0 - S_2 x^2 - S_1 x \cos \omega t$$

- Model reduces to **standard t -independent EBM** when $S_1 = 0$ [steady-state $T(x)$ no longer depends on ice physics]: **instability**.



$$f_0 \equiv aS - [A + B(T - T_m)] + D \nabla_x^2 T + F_b$$

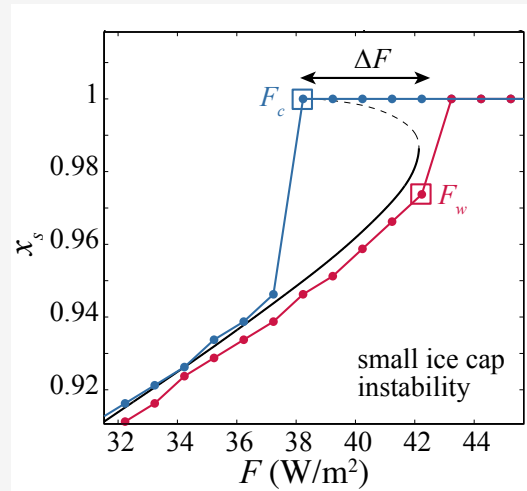
- Model reduces to **standard x -independent SCM** when $D = 0$ (no horizontal communication between $T(t)$ in each column): **instability**.

Exploring parameter space (D, S_1)

Run model with

- 21 values of S_1 in $[0, S_1^*]$
- 21 values of D in $[0, D^*]$

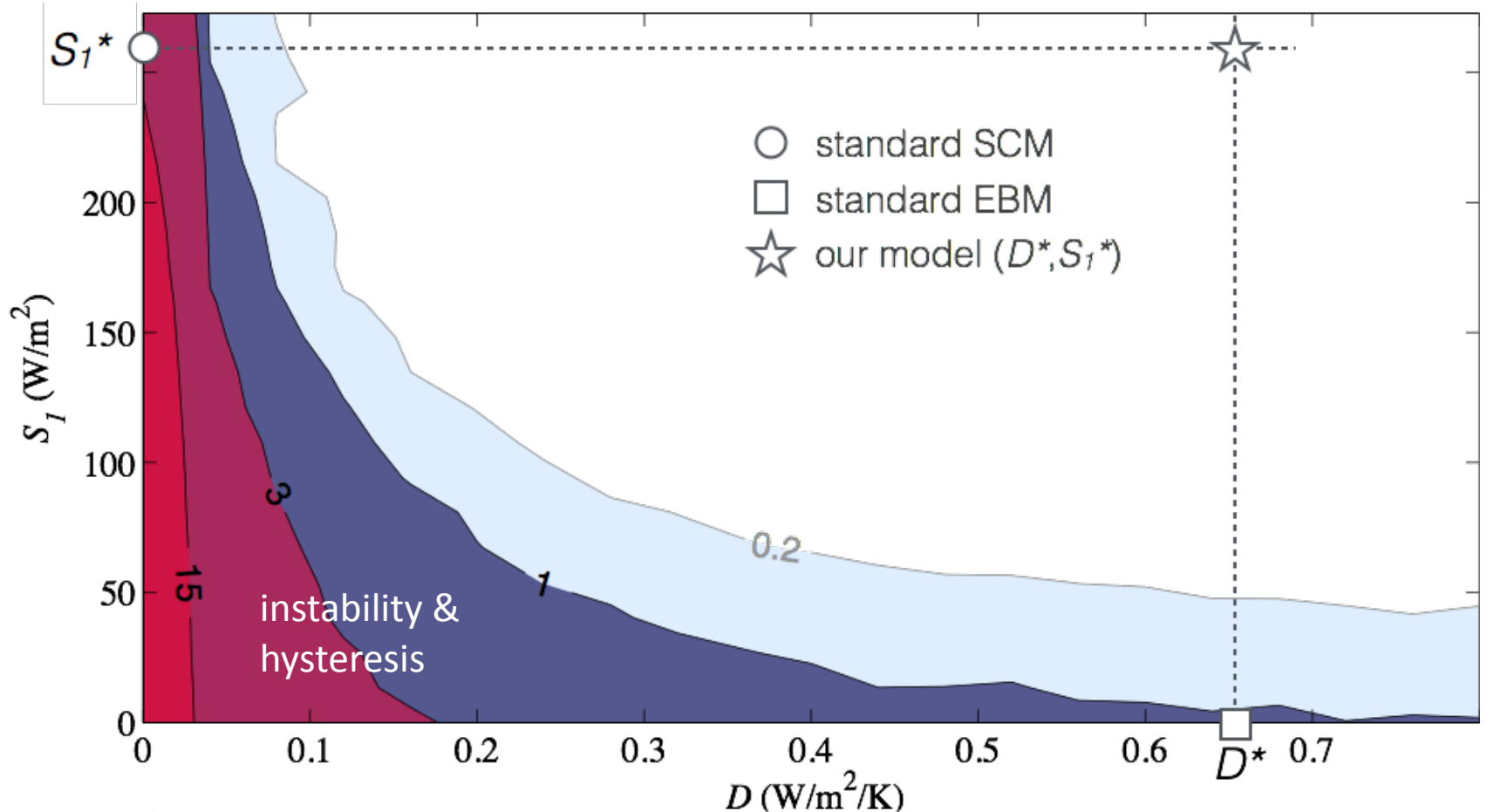
 } 441 hysteresis loop simulations.



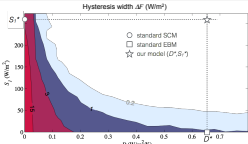
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Hysteresis width ΔF (W/m^2)



➤ Meridional communication and seasonal cycle in solar forcing **each increase stability.**



Behavior at origin

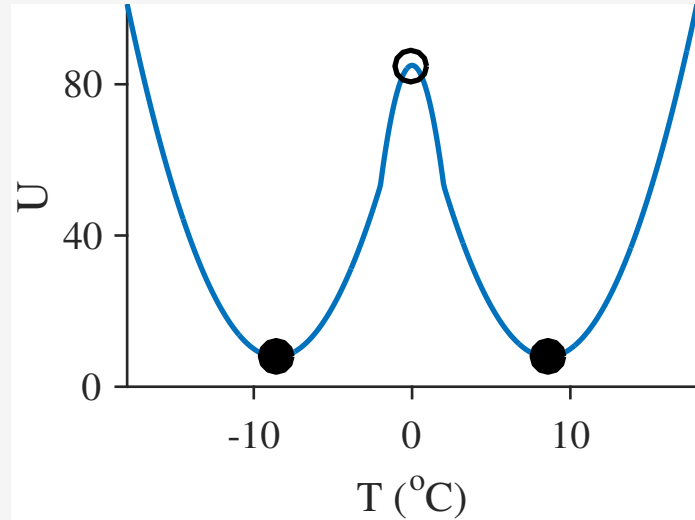
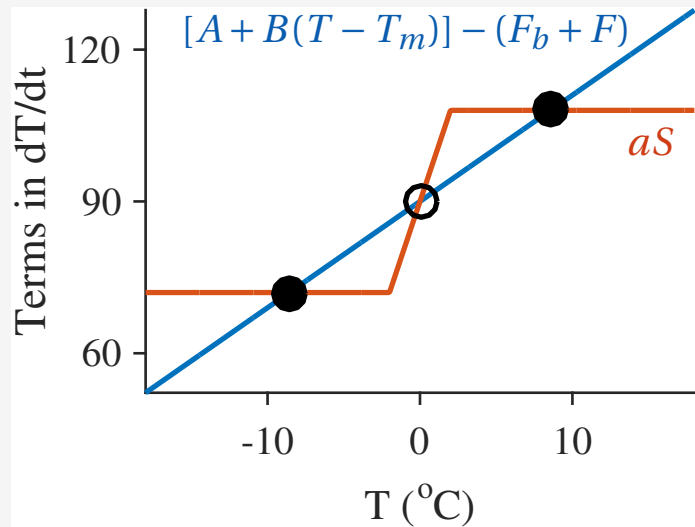
- Consider **simplest regime** ($D = 0, S_I = 0$).

$$c_w \frac{dT}{dt} = \underbrace{aS}_{\text{solar}} - \underbrace{[A + B(T - T_m)]}_{\text{OLR}} + \underbrace{F_b}_{\text{ocean heating}} + \underbrace{F}_{\text{climate forcing}}$$

- Albedo jump causes **multiple states**.

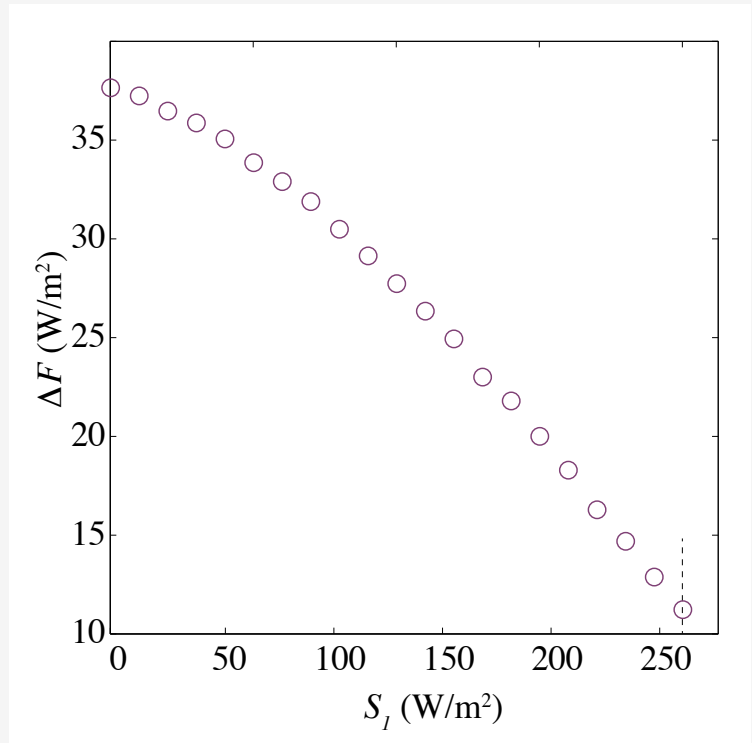
- Can visualize as “wells” of potential (U):

$$\frac{dT}{dt} = -\frac{dU}{dT}$$

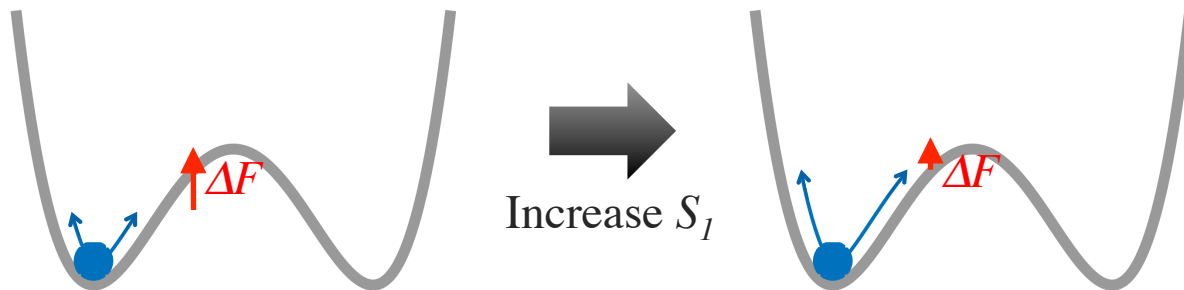


Dependence on seasonal cycle

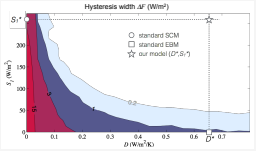
- Consider **SCM regime** ($D = 0$).
- Nearly analytical solution for $\Delta F(S_I)$ – just need numerical solution for ice thickness (cf. Eisenman 2012).
- Increasing seasonal amplitude reduces ΔF .
- Conceptually, **seasonal variations make it easier to spontaneously jump between two potential wells**, making it more difficult to support bistability.



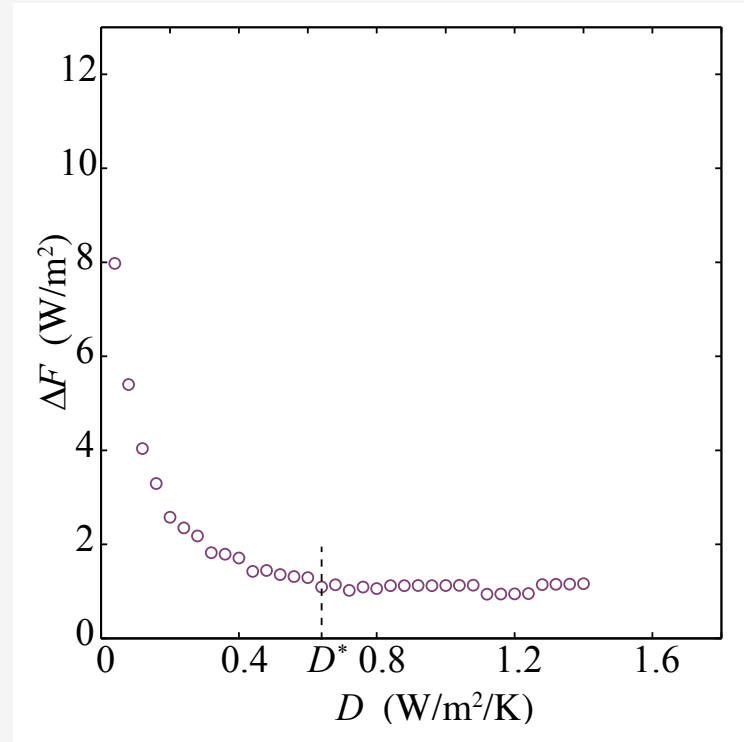
Conceptual cartoon: potential well



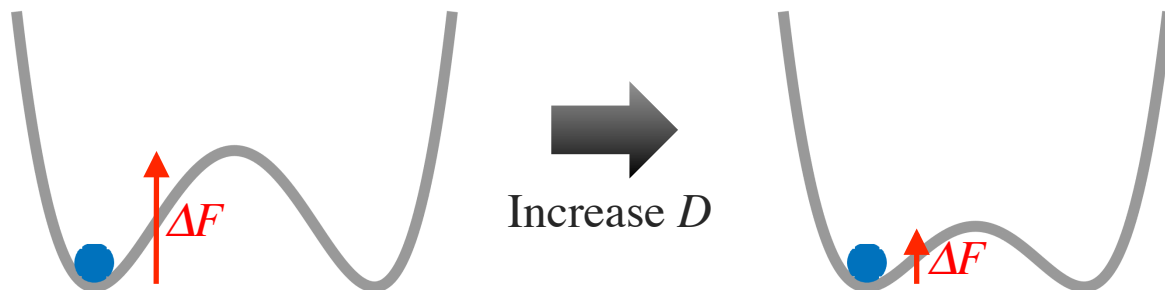
Dependence on horizontal transport



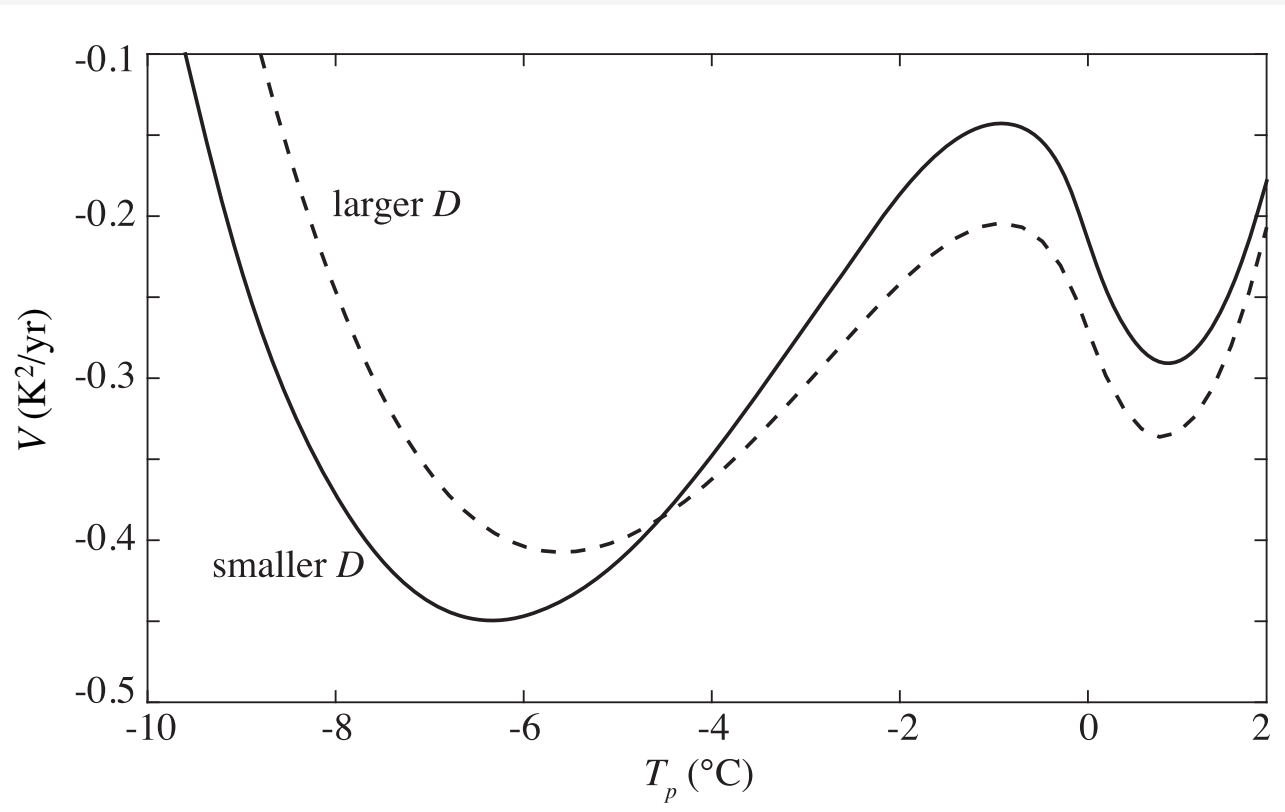
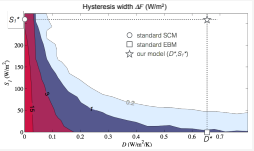
- Consider **EBM regime** ($S_I = 0$).
- $\Delta F(D)$ can be found analytically (cf. Lindzen and Farrell 1977; North 1984).
- Increasing horizontal diffusivity reduces ΔF , i.e., increases stability.
- Conceptually, **diffusion smoothes out bistability of potential well**, making it more difficult to support bistability.



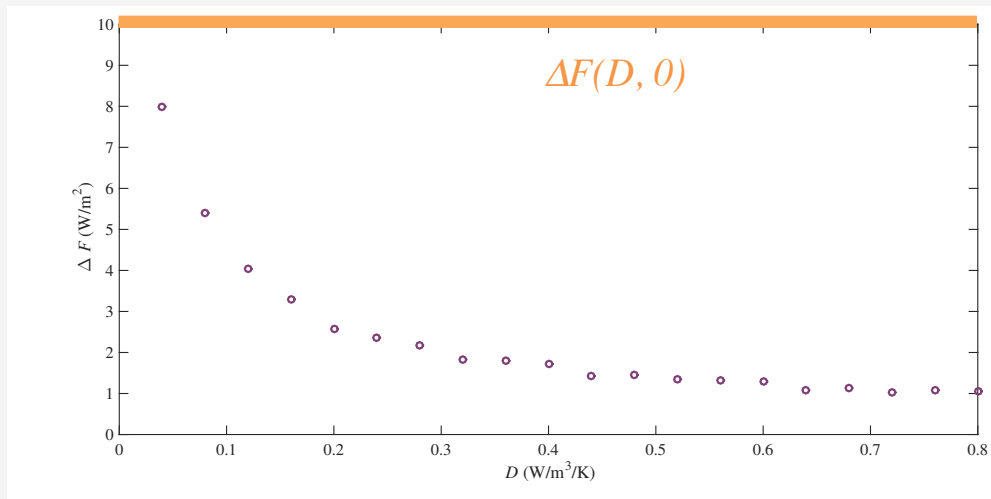
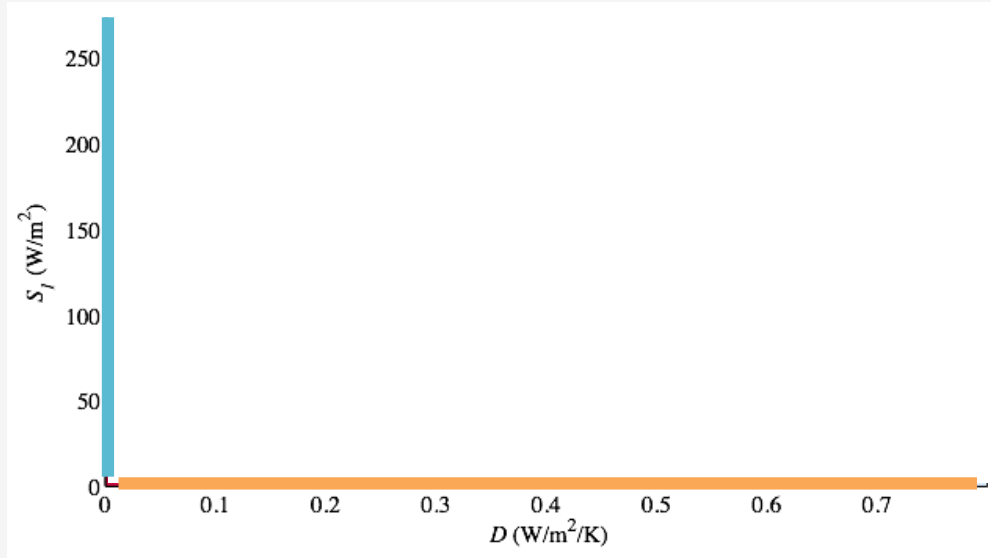
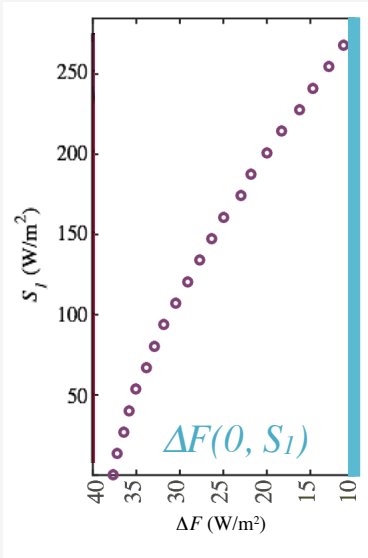
Conceptual cartoon: potential well



Dependence on horizontal transport

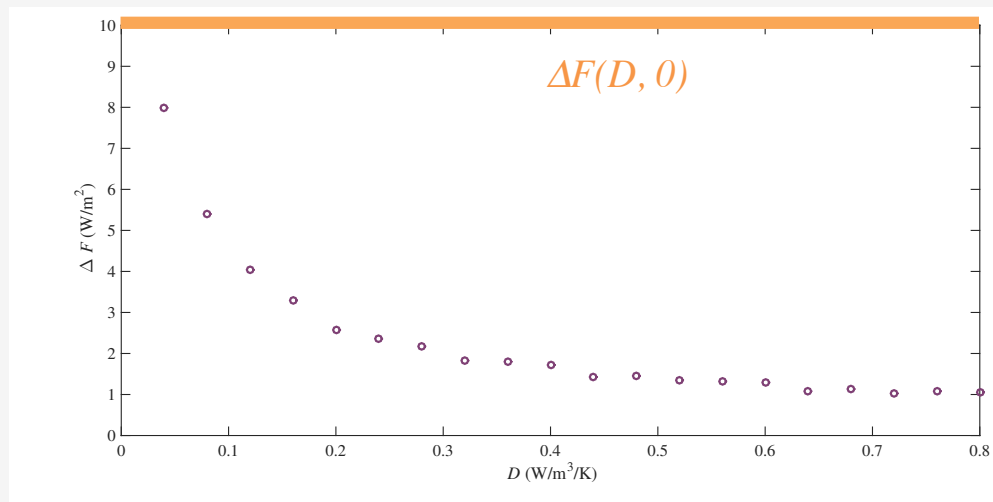
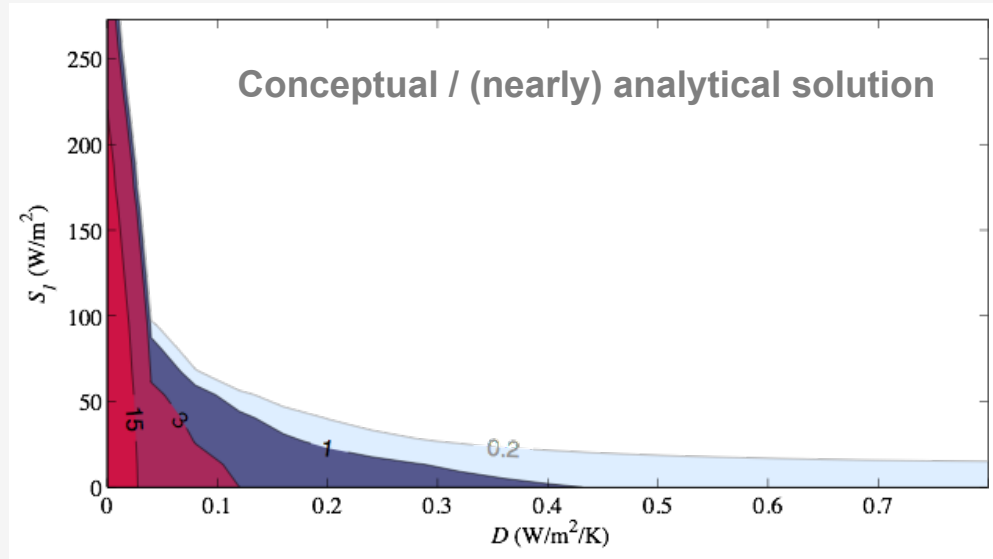
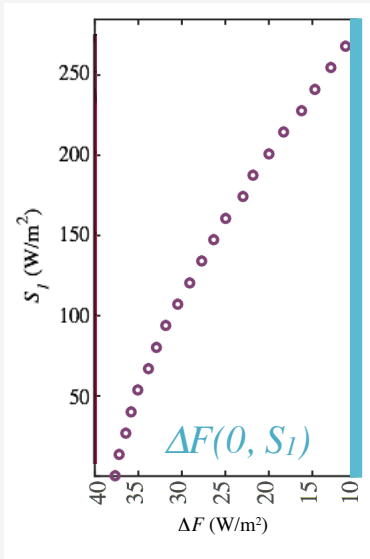


Putting it all together: $\Delta F(D, S_1)$



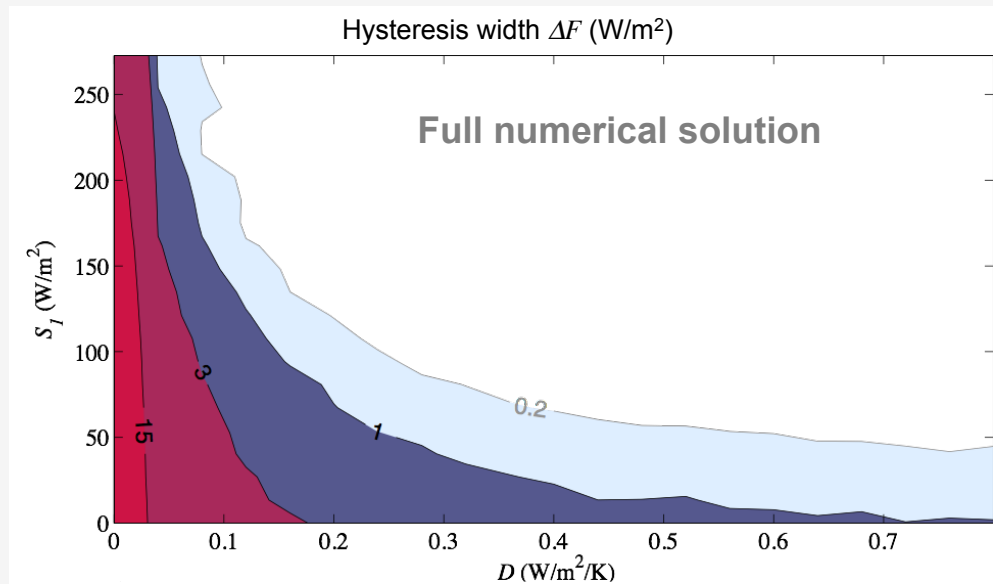
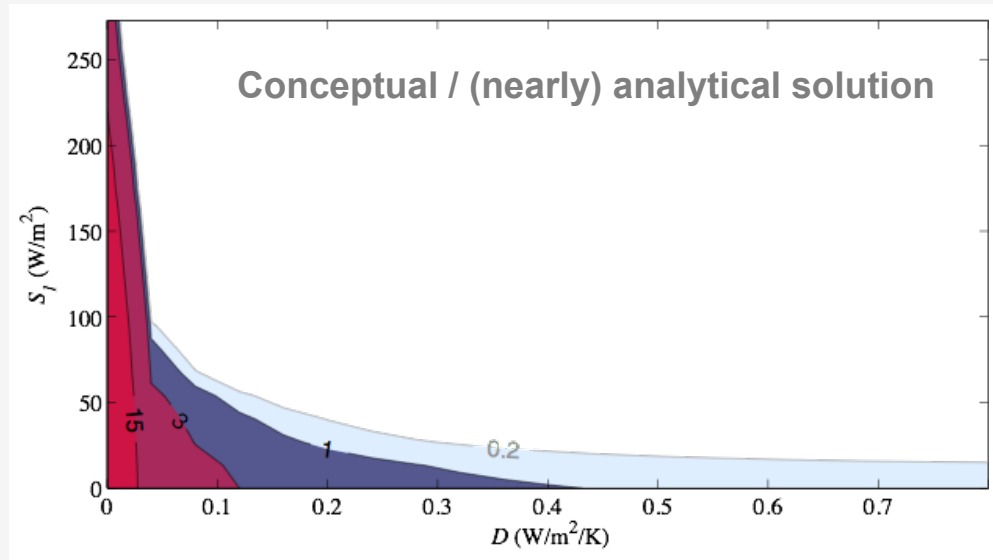
Expect that $\Delta F(D, S_1) \approx \Delta F(0, S_1) + \Delta F(D, 0)$

Putting it all together: $\Delta F(D, S_1)$



Expect that $\Delta F(D, S_1) \approx \Delta F(0, S_1) + \Delta F(D, 0)$

Putting it all together: $\Delta F(D, S_I)$



Summary of Part 1

- Why do low-order idealized models simulate instability in the sea ice cover while comprehensive GCMs do not?
- Because idealized models have typically **neglected either seasonal variations or meridional heat transport**, and both have strong stabilizing effects.
- Including both S_I and D causes ice cover to be stable.
- The **sea ice cover may be substantially more stable than has been suggested** in previous studies that used EBMs or SCMs.
- May be relevant to other cases with bistability only in low-order climate models.

Further details: **Wagner & Eisenman, J. Climate 2015**

Model code: **<http://eisenman.ucsd.edu>**

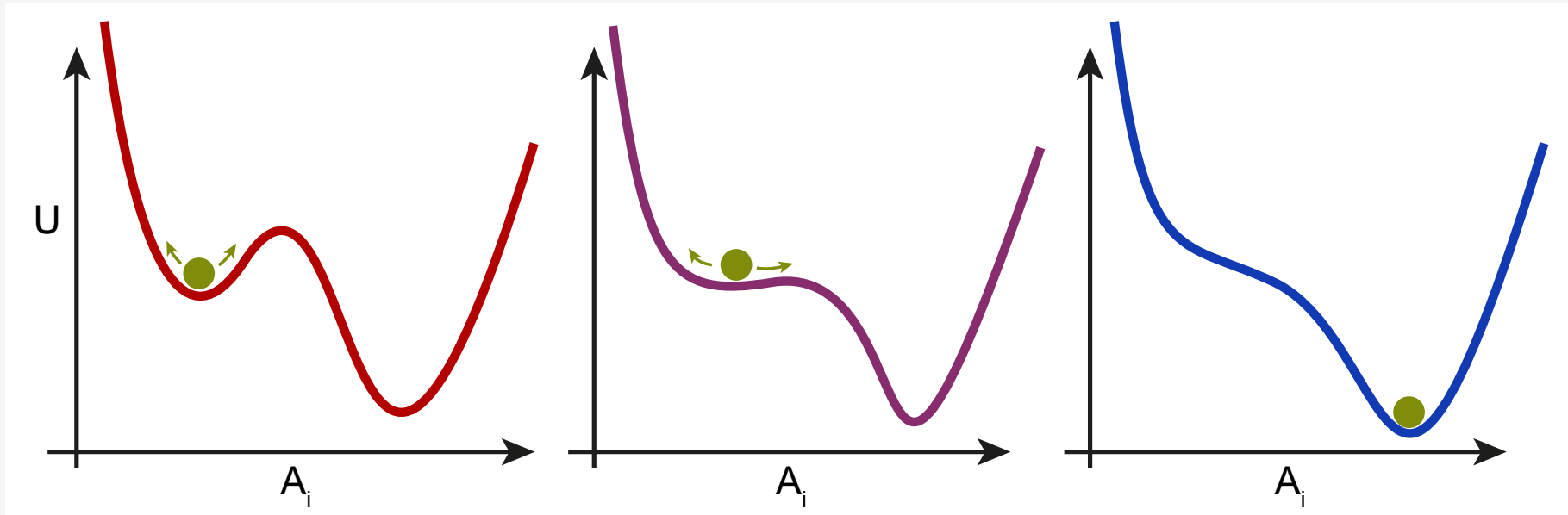
Part 2

Early warning signals

Early warning of approaching bifurcations

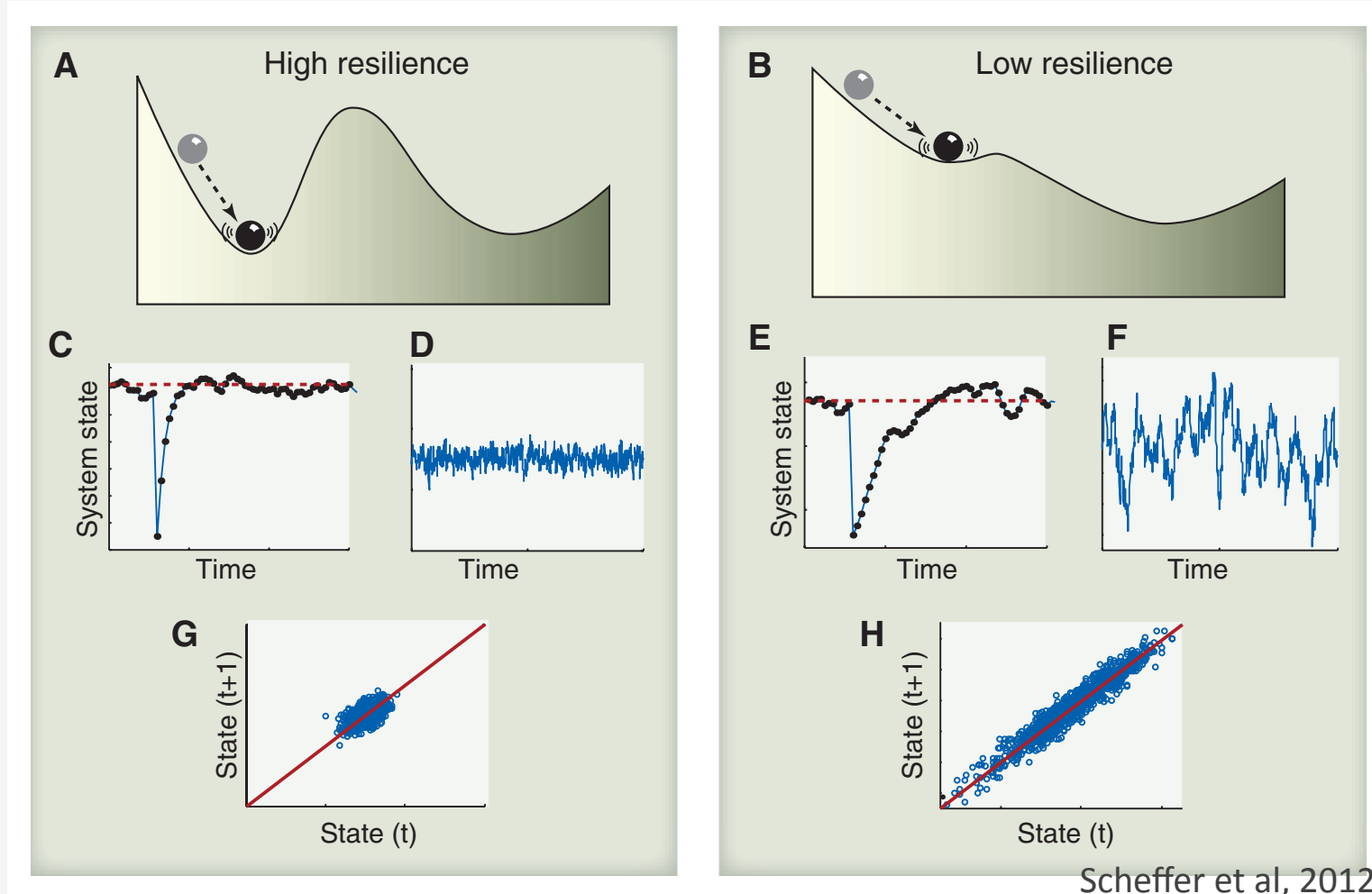
- A flurry of recent studies have investigated **early warning signals** to **identify an approaching bifurcation** before it is reached.
- The **White House** has considered using **geoengineering** to avoid crossing climate tipping points (Associated Press, 2009).

Early warning signals: critical slowing down (1/2)



- As a bifurcation is approached in a simple dynamic system, the **potential well** becomes less steep.

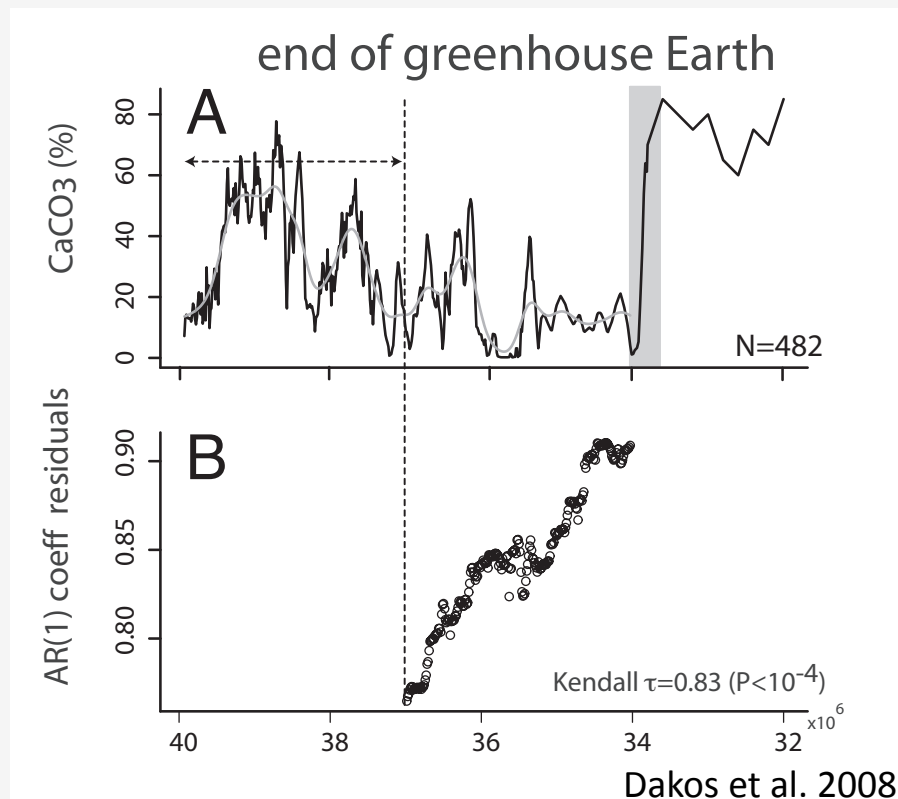
Early warning signals: critical slowing down (2/2)



- In a system subject to noise, this causes **larger autocorrelation** (i.e., slower recovery time) and **often larger variance**: “**critical slowing down**” warns of approaching bifurcation.
- **Autocorrelation is the leading candidate** to act as an early warning indicator.

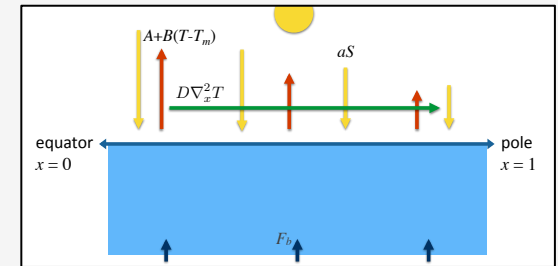
Previous studies of early warning signal

- Autocorrelation has been proposed as an early warning signal of an approaching bifurcation in a wide range of systems (e.g., Scheffer et al. 2009, Scheffer et al. 2012).
- This has been examined in paleoproxy time series and climate models (e.g., Dakos et al. 2008, Lenton et al. 2012), modern satellite observations (e.g., Livina & Lenton 2013), financial markets (e.g., Hong & Stein 2003), ecosystems (e.g., Carpenter & Brock 2006), etc...

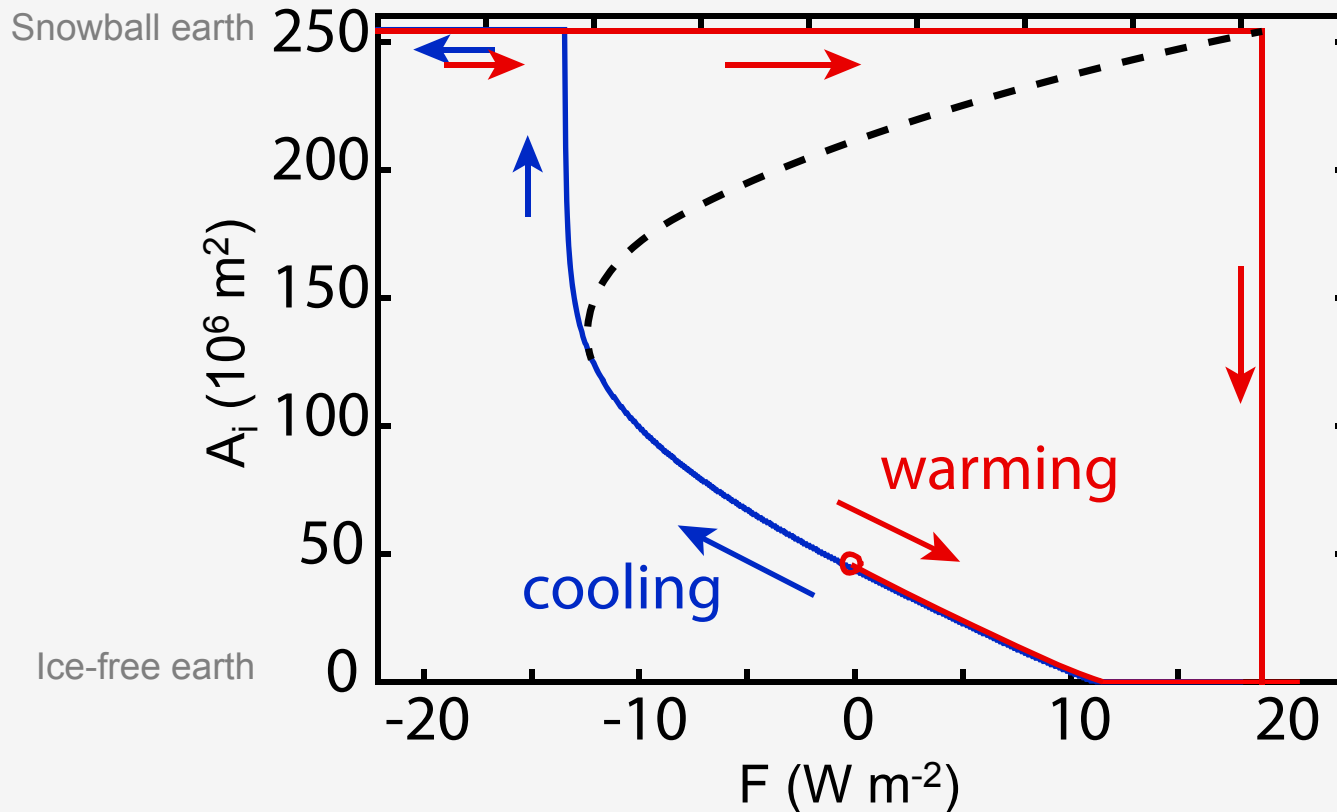


Question

- How does autocorrelation evolve in our model when we add noise and warm or cool the climate?

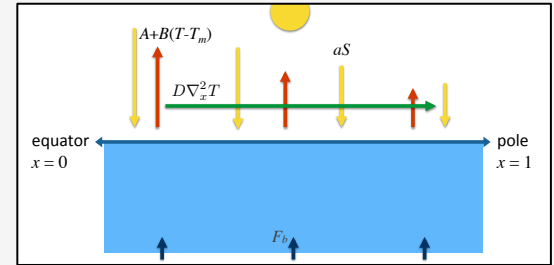


- From Part 1: This model gets no bifurcation when warmed from modern conditions, but cooling leads to snowball earth bifurcation.



Approach

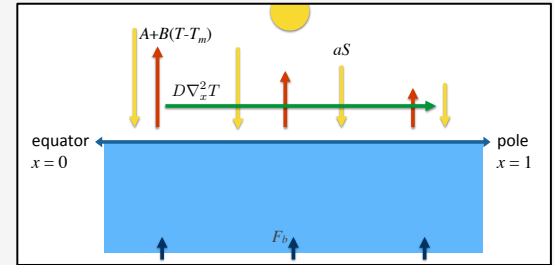
- Begin with model described in Part 1.



$$\frac{\partial E}{\partial t} = \underbrace{aS}_{\text{solar}} - \underbrace{(A + BT)}_{\text{OLR}} + \underbrace{D\nabla_x^2 T}_{\text{transport}} + \underbrace{F_b}_{\text{ocean heating}} + \underbrace{F}_{\text{climate forcing}}$$

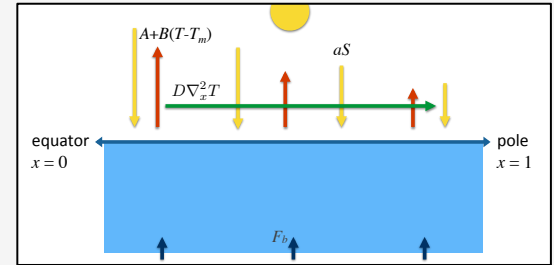
Approach

- Begin with model described in Part 1.
- Add (weather) noise.



$$\frac{\partial E}{\partial t} = \underbrace{aS}_{\text{solar}} - \underbrace{(A + BT)}_{\text{OLR}} + \underbrace{D\nabla_x^2 T}_{\text{transport}} + \underbrace{F_b}_{\text{ocean heating}} + \underbrace{F}_{\text{climate forcing}} + \underbrace{N}_{\text{noise}}$$

Approach



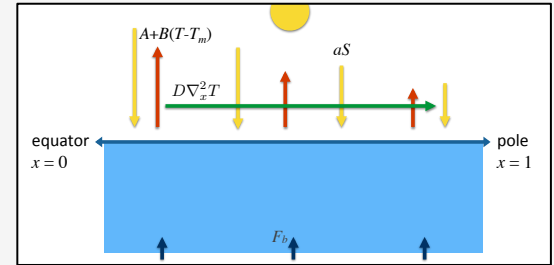
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(Deal with complications associated with numerical integration...)

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Approach



- Begin with model described in Part 1.

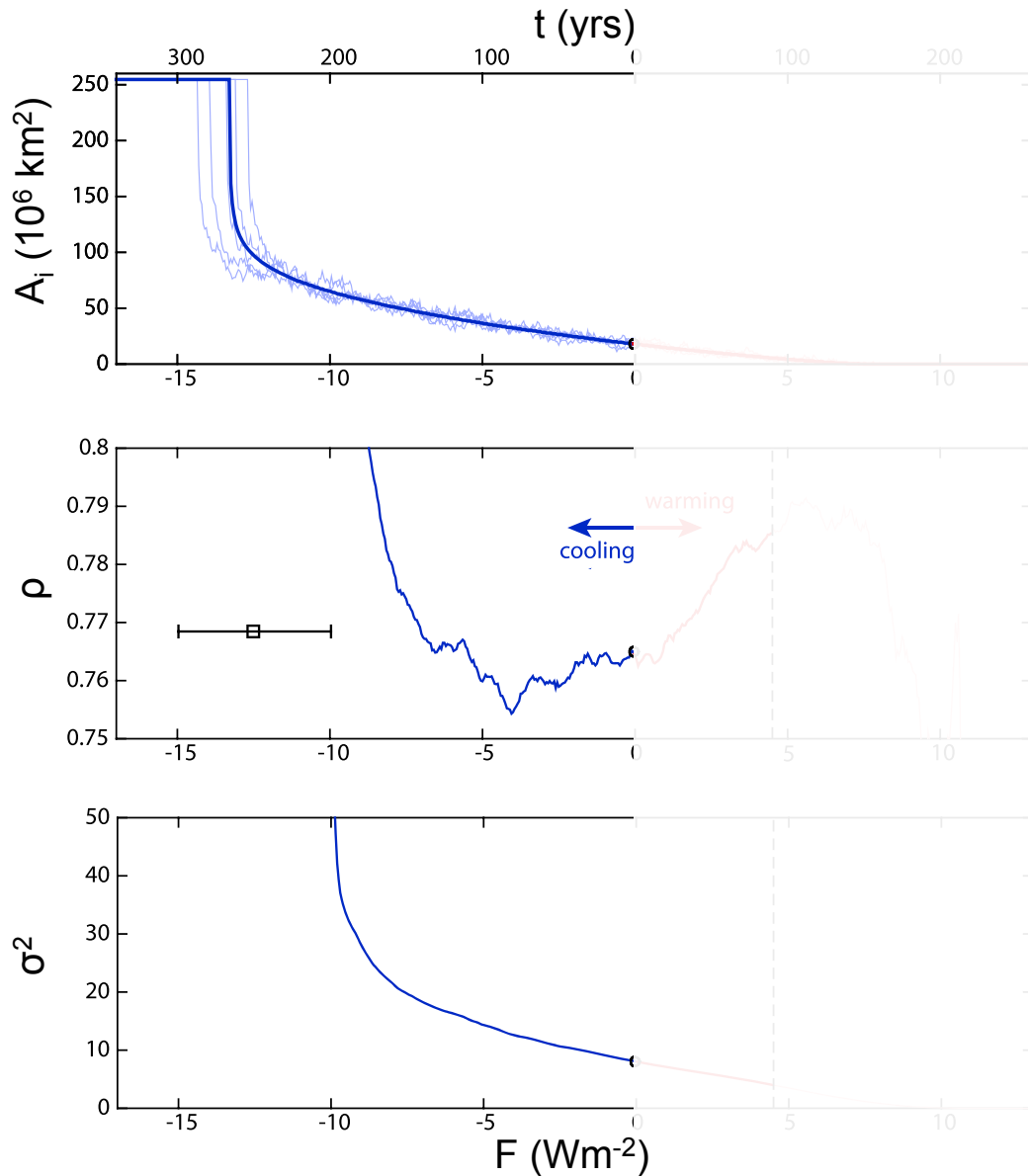
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- Compute 10,000 realizations of noisy warming and cooling (varying F).
- Focus on September sea ice area.

Results: Cooling to Snowball earth

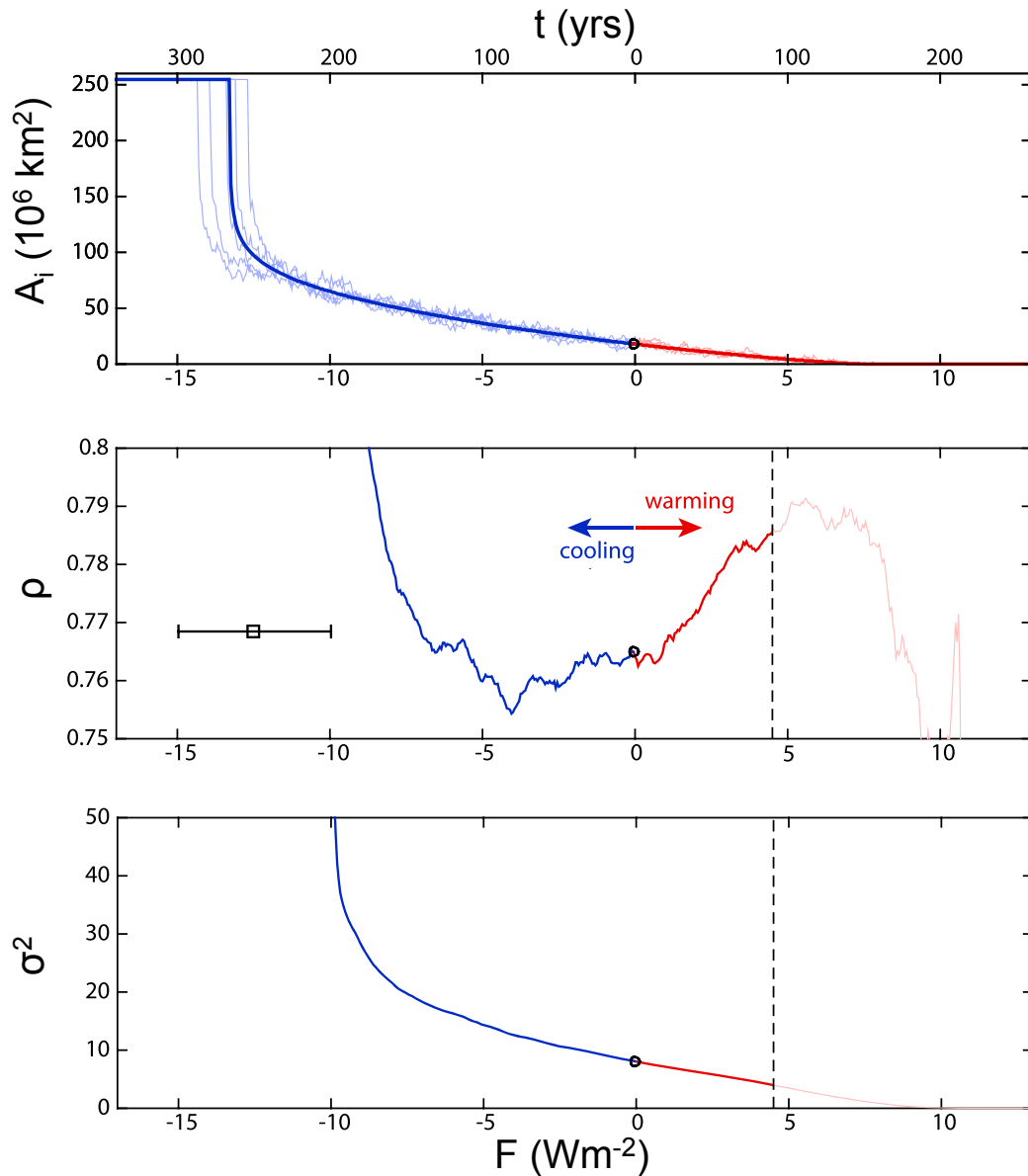


- Climate cools until bifurcation, then **abruptly jumps** to Snowball earth.

- Lag-1yr **autocorrelation increases**. (Variance also increases).

- **Critical slowing down correctly warns** of approaching Snowball earth bifurcation.

Results: Warming to ice-free earth



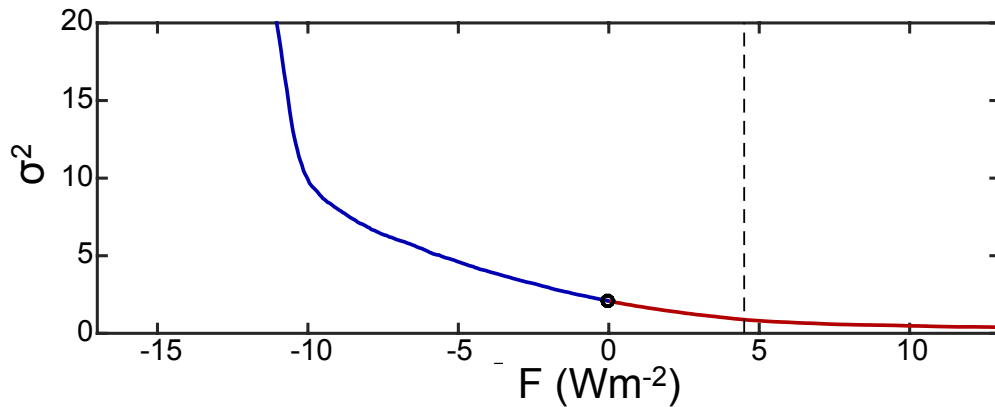
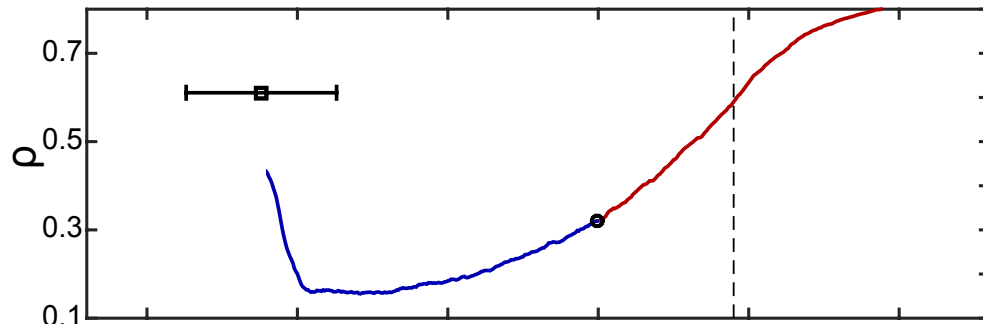
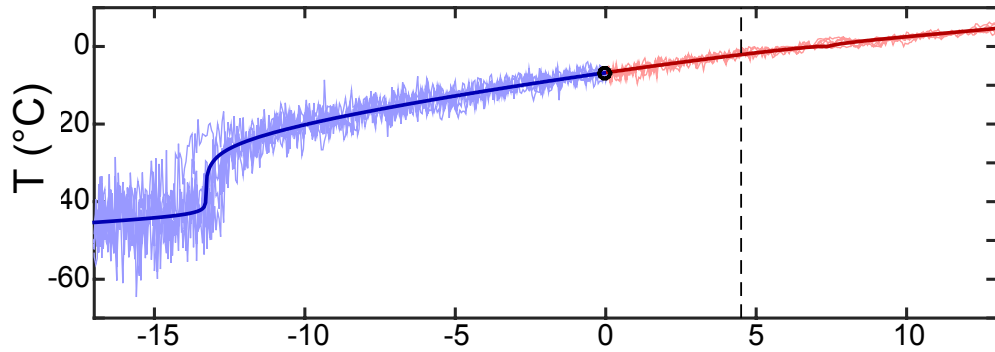
- Ice declines smoothly, with **no acceleration or abrupt loss.**

- Lag-1yr **autocorrelation increases.** (Variance, however, decreases.)

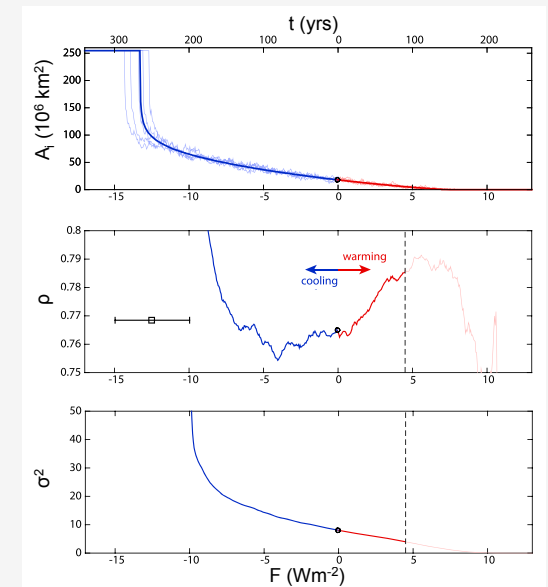
➤ **Critical slowing down raises a false alarm!**

It warns of a bifurcation that is not actually there.

False alarm mechanism: Temperature at pole



- Temperature at pole shows features that are qualitatively equivalent to the sea ice area.



False alarm mechanism: Simplified model

$$\frac{\partial E}{\partial t} = \underbrace{a(E) S(t)}_{\text{solar}} - \underbrace{(A + BT)}_{\text{OLR}} + \underbrace{D\nabla_x^2 T}_{\text{transport}} + \underbrace{F_b}_{\text{ocean heating}} + \underbrace{F}_{\text{climate forcing}} + \underbrace{N}_{\text{noise}}$$

- Part 1 results suggest that seasonal variations and heat transport act to reduce the effect of nonlinearity from albedo changes.

➤ **Removing transport & seasonal cycle** while using **constant albedo** may *plausibly* have compensating effects, with results qualitatively unaffected.

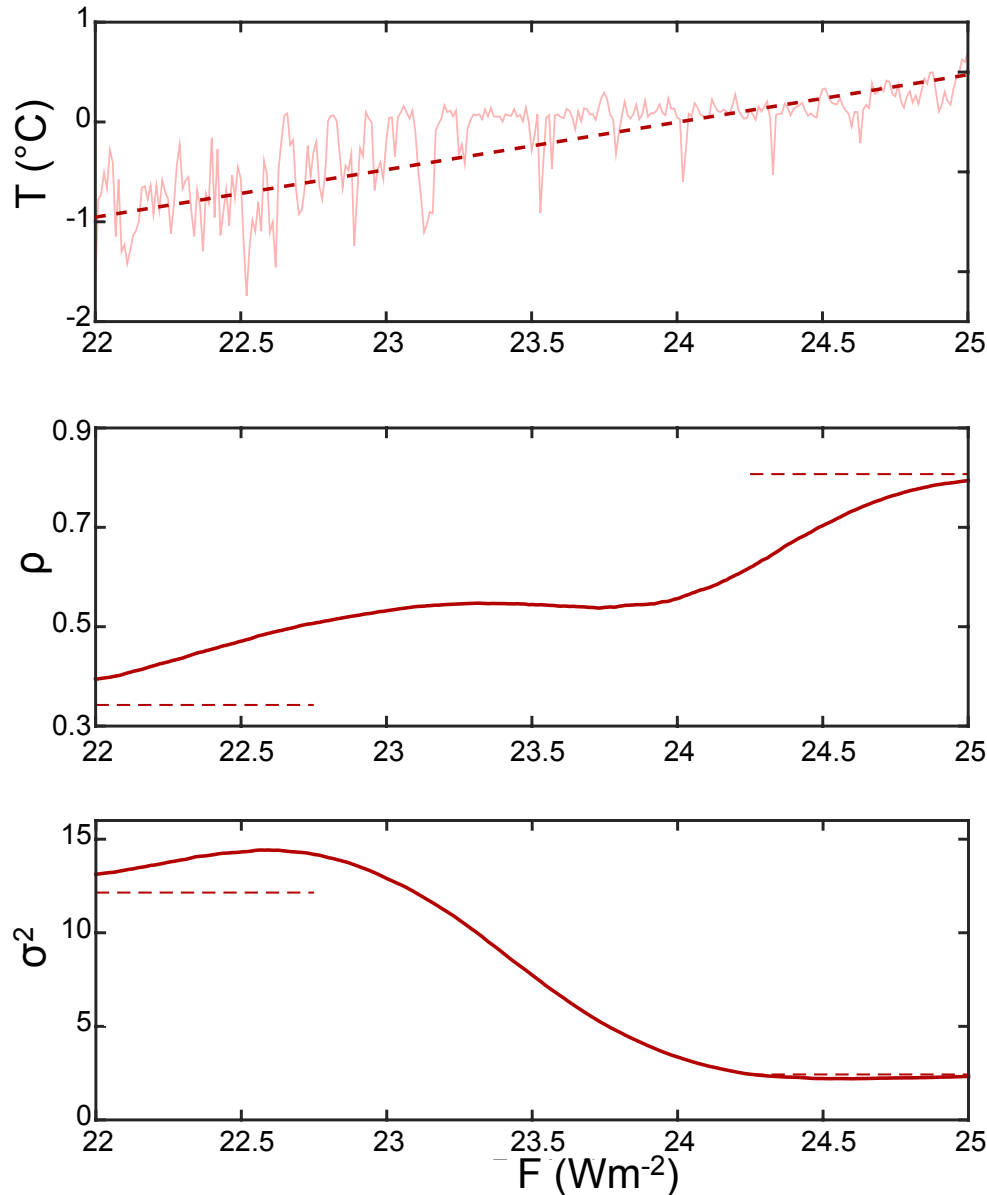
$$\frac{\partial E}{\partial t} = \underbrace{a_0 \bar{S}}_{\text{solar}} - \underbrace{(A + BT)}_{\text{OLR}} + \underbrace{F_b}_{\text{ocean heating}} + \underbrace{F}_{\text{climate forcing}} + \underbrace{N}_{\text{noise}} = \alpha - BT + N$$

- In this simplified system, influence of sea ice thermodynamics can be crudely approximated as a **change in effective heat capacity**:

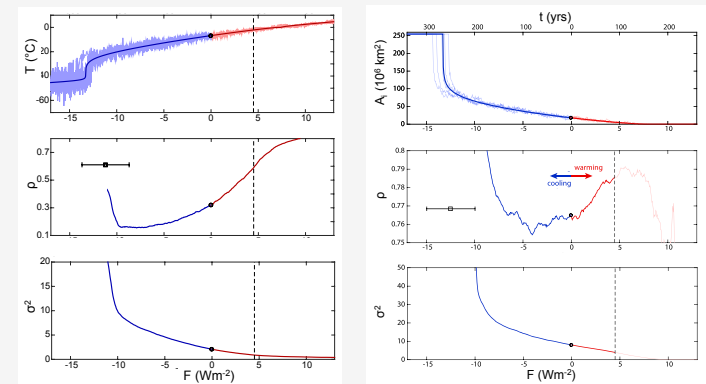
$$c(T) \frac{\partial T}{\partial t} = \alpha - BT + N \quad c(T) \equiv \begin{cases} c_w/5 & T < 0 \\ c_w & T > 0 \end{cases}$$

- This system represents an Ornstein-Uhlenbeck process (analytically solvable).

False alarm mechanism: Simplified model results

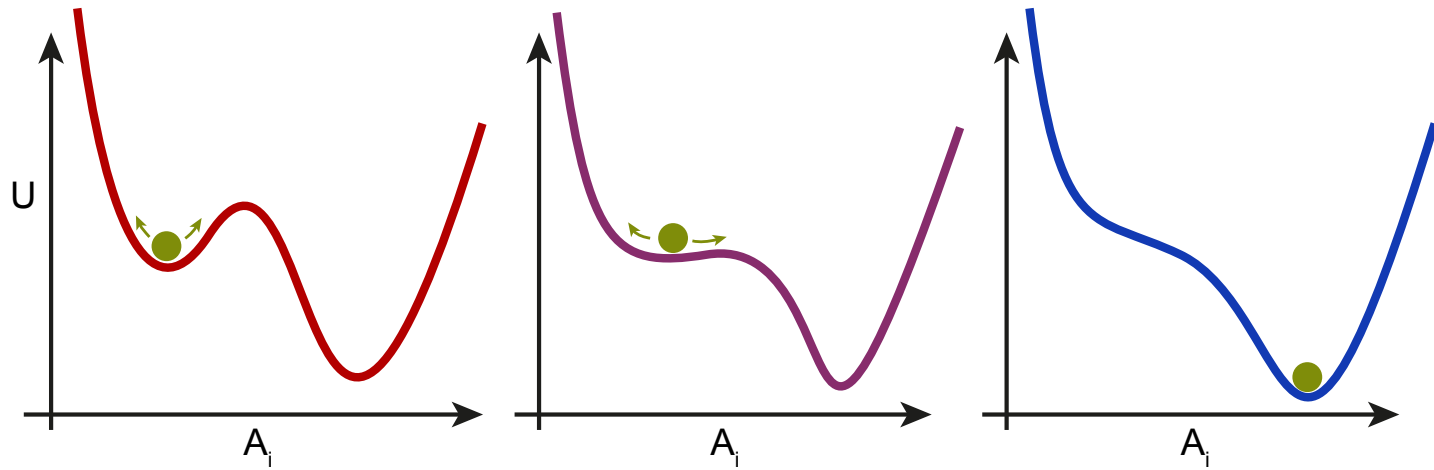


- The simplified model results resemble the full model.
- Suggests simple explanation: the **increase in effective heat capacity** when sea ice is replaced with open ocean **causes autocorrelation to increase** (while the variance decreases).
- No bifurcation or abrupt change occurs.

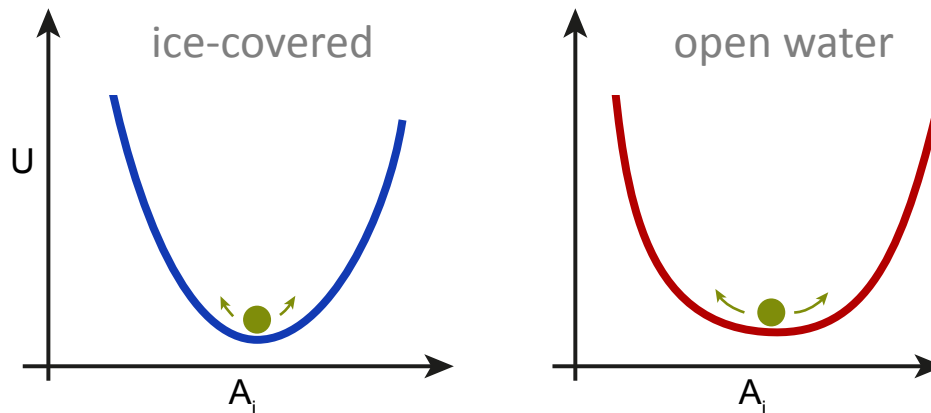


Take home message: don't extrapolate

Standard early warning signals



Sea ice loss in this model



Summary of Part 2

- Rising autocorrelation is the leading candidate to act as an early warning signal for abrupt change.
- Our model has no abrupt sea ice loss, but the autocorrelation nonetheless increases during warming.
- This slowing down appears to be due to a **change in the effective heat capacity** from *ice-covered* (fast response) to *open water* (slow response).
- **Early warning signals can raise false alarms** during sea ice retreat, warning of bifurcations that are not actually there.

Further details: **Wagner & Eisenman, GRL 2015**

Summary of Part 1

- Why do low-order idealized models simulate instability in the sea ice cover while comprehensive GCMs do not?
- Because idealized models have typically **neglected either seasonal variations or meridional heat transport**, and both have strong stabilizing effects.
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